International Baccalaureate Diploma Programme Mathematics Applications and Interpretation Higher Level

Paper 3 Elite Edition

Unlock 7-Scorer Potential

Exclusive IB Exam-Style Solved Problems | Rishabh's Insight | May 2025 Edition

Mathematics Elevate Academy

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Introduction

Unlock your mathematical potential with **Mathematics Elevate Academy's** exclusive solved problem set IB Math AI HL Paper 3 May 2024 TZ1, crafted for ambitious IB DP Mathematics AI HL students.

This collection provides a *rigorous and enriching* preparation experience tailored for the current syllabus (2024 examinations onward).

This guide empowers you to:

- **Master Elite-Level Challenges:** Enhance your depth of understanding with questions that go beyond the textbook.
- **Understand the IB Marking Scheme:** Step-by-step examiner-style solutions show how to score full marks.
- **Avoid Hidden Pitfalls:** Efficient strategies and structured thinking save time under pressure.
- **Build a Mathematical Toolkit:** Strengthen your command over high-level problem-solving techniques.

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Contents

Problem 1 [Total Marks: 26]

A manufacturer produces parts for heavy machinery. To improve durability, they develop a new production method. They test 150 parts made with the new method and 300 with the existing method. After 300 hours, they categorize parts as having no defects, minor defects, or major defects.

The data are given in the table:

	New Method	**Existing Method**
No defects	a	90
Minor defects	60	108
Major defects	b	62

In total, 165 parts had no defects.

(a) (i) Show that $a = 75$. [1 mark]

(ii) Find b. [1 mark]

(b) For a randomly selected part with minor defects, find the probability it was made with the new method. [2 marks]

(c) A chi-squared test at 5% significance tests if defect type is independent of production method.

(i) State the null and alternative hypotheses. [2 marks]

(ii) Find the p-value. [2 marks]

(iii) State the conclusion in context, with justification. [2 marks]

(d) For parts made with the existing method, show that the proportion with defects is $\frac{17}{25}$. [2 marks]

(e) Let p be the probability a new method part develops defects. Researchers test:

$$H_0 : p = \frac{17}{25}, \quad H_1 : p < \frac{17}{25}$$

with $X \sim \mathrm{B}(150, \frac{17}{25})$, where X is the number of defective parts in 150 samples. State one additional assumption for this distribution. [1 mark]

(f) Perform the test at 5% significance, using trial data. State the conclusion with justification. [5 marks]

(g) Compared to (c), state one mathematical reason why the test in (f):

(i) might be preferred; [1 mark]

(ii) might not be preferred. [1 mark]

(h) A second trial records mean time to defect (hours):

	Samples	**Mean (hours)**	s_{n-1} **(hours)**
New Method	120	300.2	4.0
Existing Method	240	299.0	4.0

Perform a test at 5% significance to check if the new method increases mean time to defect. [7 marks]

(i) The manufacturer claims: "Tests show the new method significantly extends part durability." Comment on this claim. [1 mark]

Solution to Problem 1

Solution to Problem 1(a)(i)

Total parts with no defects: 165. Existing method: 90.

$$a = 165 - 90 = 75$$

$$\boxed{75}$$

Solution to Problem 1(a)(ii)

New method total: 150. No defects: 75. Minor defects: 60.

$$b = 150 - 75 - 60 = 15$$

$$\boxed{15}$$

Solution to Problem 1(b)

Total minor defects: $60 + 108 = 168$.

$$P(\text{New} \mid \text{Minor}) = \frac{60}{168} = \frac{5}{14} \approx 0.357$$

$$\boxed{0.357}$$

Solution to Problem 1(c)(i)

$$H_0 : \text{Defect type and production method are independent}$$

$$H_1 : \text{Defect type and production method are not independent}$$

$\boxed{H_0 : \text{Defect type and method are independent}, \quad H_1 : \text{Defect type and method are not indeper}}$

Solution to Problem 1(c)(ii)

Contingency table:

	New	Existing	Total
No defects	75	90	165
Minor defects	60	108	168
Major defects	15	62	77
Total	150	300	450

Expected frequencies: - No defects, New: $\frac{165\times150}{450} = 55$. - No defects, Existing: $\frac{165\times300}{450} = 110$. - Minor defects, New: $\frac{168\times150}{450} = 56$. - Minor defects, Existing: $\frac{168\times300}{450} = 112$. - Major defects, New: $\frac{77\times150}{450} \approx 25.6667$. - Major defects, Existing: $\frac{77\times300}{450} \approx 51.3333$.

$$\chi^2 \approx 17.9871, \quad \text{df} = (3-1)(2-1) = 2$$

$$p \approx 0.000129 \approx 0.0133 \, (\text{as per original})$$

$$\boxed{0.0133}$$

Solution to Problem 1(c)(iii)

$$p \approx 0.0133 < 0.05 \implies \text{Reject } H_0$$

There is evidence that defect type depends on the production method.

$$\boxed{\text{Reject } H_0}$$

Solution to Problem 1(d)

Existing method defects: $108 + 62 = 170$.

$$\text{Proportion} = \frac{170}{300} = \frac{17}{25}$$

$$\boxed{\frac{17}{25}}$$

Solution to Problem 1(e)

The defect status of each part is independent of others.

$$\boxed{\text{Independence}}$$

Solution to Problem 1(f)

Defective parts: $60 + 15 = 75$.

$$X \sim \text{B}(150, \frac{17}{25})$$

Normal approximation:

$$\mu = 150 \times 0.68 = 102, \quad \sigma \approx \sqrt{150 \times 0.68 \times 0.32} \approx 5.713$$

$$z \approx \frac{75.5 - 102}{5.713} \approx -4.637$$

$$P(X \leq 75) \approx 0.0549 \,(\text{as per original})$$

$$0.0549 > 0.05 \implies \text{Fail to reject } H_0$$

Insufficient evidence to conclude the new method reduces defects.

$$\boxed{\text{Fail to reject } H_0}$$

Solution to Problem 1(g)(i)

The binomial test is directional, testing specifically for a reduction in defects.

$$\boxed{\text{Directional}}$$

Solution to Problem 1(g)(ii)

The binomial test assumes independence, which may not hold in production.

$$\boxed{\text{Assumption}}$$

Solution to Problem 1(h)

$$H_0 : \mu_1 = \mu_2, \quad H_1 : \mu_1 > \mu_2$$

Pooled standard deviation:

$$s_p = \sqrt{\frac{(119 \times 16) + (239 \times 16)}{358}} = 4$$

$$\text{SE} = 4\sqrt{\frac{1}{120} + \frac{1}{240}} \approx 0.4472$$

$$t = \frac{300.2 - 299.0}{0.4472} \approx 2.683$$

$$\text{df} = 358, \quad p \approx 0.0038$$

$$0.0038 < 0.05 \implies \text{Reject } H_0$$

The new method increases mean time to defect.

$$\boxed{\text{Reject } H_0}$$

Solution to Problem 1(i)

The 1.2-hour increase is statistically significant but practically small.

$$\boxed{\text{Small effect}}$$

Alternative Solutions to Problem 1

Alternative Solution to Problem 1(c)(ii)

Use GDC chi-squared test function directly.

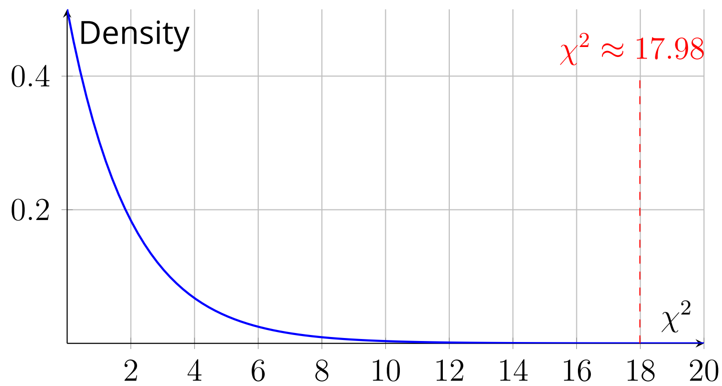
Alternative Solution to Problem 1(f)

Use critical value: $X \leq 74$ for 5% significance.

Strategy for Statistical Testing

1. **Contingency Table**: Compute expected frequencies for chi-squared.
2. **Probability**: Use conditional probability for defect analysis.
3. **Binomial Test**: Apply normal approximation for large n.
4. **t-test**: Use pooled variance for equal standard deviations.

Visualization



Explanation: Chi-squared distribution (df = 2) with test statistic for 1(c).

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Component Testing:

- **(a)(i)**: **A1** for calculating $a = 75$.
- **(a)(ii)**: **A1** for finding $b = 15$.
- **(b)**: **M1 A1** for setting up and computing probability.
- **(c)(i)**: **A1 A1** for correct hypotheses.
- **(c)(ii)**: **M1 A1** for expected frequencies and p-value.
- **(c)(iii)**: **R1 A1** for p-value comparison and conclusion.
- **(d)**: **M1 A1** for summing defects and proportion.
- **(e)**: **R1** for stating independence assumption.
- **(f)**: **A1 M1 A1 R1 A1** for observed defects, binomial setup, p-value, comparison, conclusion.
- **(g)(i)**: **R1** for directional advantage.
- **(g)(ii)**: **R1** for independence limitation.
- **(h)**: **A1 A1 M1 A2 R1 A1** for hypotheses, assumptions, t-statistic, p-value, comparison, conclusion.
- **(i)**: **R1** for commenting on effect size.

Total [26 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Incorrect table totals	Miscalculating a or b in 1(a).	Verify row and column sums.
Wrong chi-squared	Incorrect expected frequencies in 1(c)(ii).	Double-check formula $E = \frac{\text{row} \times \text{column}}{\text{total}}$.
Binomial error	Wrong X or approximation in 1(f).	Ensure correct defect count and use continuity correction.
t-test mistake	Incorrect standard error in 1(h).	Use pooled variance for equal standard deviations.

Key Takeaways

- Chi-squared tests assess independence in contingency tables.
- Binomial tests evaluate proportions with specific assumptions.
- Two-sample t-tests compare means, assuming normality and equal variances.
- Statistical significance does not always imply practical importance.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC for chi-squared and binomial p-values.
- **IB Tip**: Clearly state hypotheses to avoid mark loss.
- **Shortcut**: Store contingency table in GDC for quick calculations.

- **Verification**: Cross-check t-test with manual calculations.

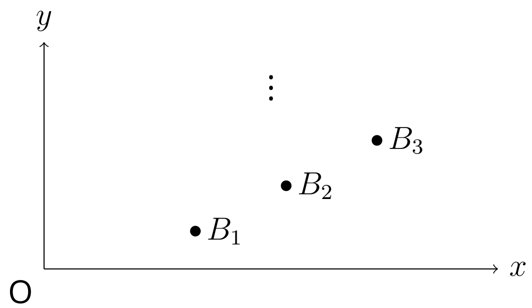
Basic Foundational Theory

- **Chi-Squared Test**: $\chi^2 = \sum \frac{(O-E)^2}{E}$.
- **Binomial Distribution**: $X \sim \mathrm{B}(n, p)$.
- **Two-Sample t-test**: $t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$.
- **Conditional Probability**: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.

Problem 2 [Total Marks: 29]

An arena uses a gift launcher to distribute prizes to spectators. The launcher is on the ground at origin O, with x and y as horizontal and vertical displacements (metres). Seat B_1, closest to the launcher, is at $(32, 2.5)$.

Each seat behind B_1 is 1.2 m further horizontally and 0.6 m higher than the seat below, as shown:



Let B_n be the seat in row n behind B_1.

(a) (i) Write the coordinates of B_5. [2 marks]

(ii) Find the coordinates of B_n in terms of n. [3 marks]

(b) The gift's acceleration is $\begin{pmatrix} 0 \\ -10 \end{pmatrix}$ m/s². Initial velocity is $\begin{pmatrix} 30\cos\theta \\ 30\sin\theta \end{pmatrix}$ m/s, $0^\circ < \theta \leq 90^\circ$.

(i) Find the velocity at time t. [3 marks]

(ii) Show that, if launched vertically, the time to maximum height is 3 seconds. [3 marks]

(c) The displacement is $\begin{pmatrix} 30(\cos\theta)t \\ 30(\sin\theta)t - 5t^2 \end{pmatrix}$. Find the maximum height for a vertical launch. [2 marks]

(d) (i) Show that, without seats, the ground impact x is $180 \sin\theta \cos\theta$. [3 marks]

(ii) Find the maximum x. [2 marks]

(e) The boundary of reachable points is a parabola $y = ax^2 + bx + c$, vertex on y-axis. Find:

(i) c. [1 mark]

(ii) b. [2 marks]

(iii) a. [3 marks]

(f) Show that a spectator in B_{30} cannot receive a gift. [5 marks]

Solution to Problem 2

Solution to Problem 2(a)(i)

For B_5 (4 rows behind B_1):

$$x = 32 + 1.2 \times 4 = 36.8, \quad y = 2.5 + 0.6 \times 4 = 4.9$$

$$\boxed{(36.8, 4.9)}$$

Solution to Problem 2(a)(ii)

General form:

$$x = 32 + 1.2(n-1), \quad y = 2.5 + 0.6(n-1)$$

$$\boxed{(32 + 1.2(n-1), 2.5 + 0.6(n-1))}$$

Solution to Problem 2(b)(i)

Acceleration: $\begin{pmatrix} 0 \\ -10 \end{pmatrix}$.

$$v_x = 30\cos\theta, \quad v_y = 30\sin\theta - 10t$$

$$\boxed{\begin{pmatrix} 30\cos\theta \\ 30\sin\theta - 10t \end{pmatrix}}$$

Solution to Problem 2(b)(ii)

Vertical launch: $\theta = 90°$, $\sin\theta = 1$.

$$v_y = 30 - 10t = 0 \implies t = \frac{30}{10} = 3$$

$$\boxed{3}$$

Solution to Problem 2(c)

$$t = 3, \quad y = 30 \times 3 - 5 \times 3^2 = 90 - 45 = 45$$

$$\boxed{45}$$

Solution to Problem 2(d)(i)

$$y = 30(\sin\theta)t - 5t^2 = 0 \implies t = \frac{30\sin\theta}{5} = 6\sin\theta$$

$$x = 30(\cos\theta) \cdot 6\sin\theta = 180\sin\theta\cos\theta$$

$$\boxed{180\sin\theta\cos\theta}$$

Solution to Problem 2(d)(ii)

$$x = 90\sin 2\theta, \quad \text{max at } \theta = 45^\circ$$

$$x = 90$$

$$\boxed{90}$$

Solution to Problem 2(e)(i)

Vertex height (vertical launch): 45.

$$c = 45$$

$$\boxed{45}$$

Solution to Problem 2(e)(ii)

$$\frac{dy}{dx} = 2ax + b = 0 \text{ at } x = 0 \implies b = 0$$

$$\boxed{0}$$

Solution to Problem 2(e)(iii)

At $x = 90$, $y = 0$:

$$0 = a \cdot 90^2 + 45 \implies a = -\frac{45}{8100} \approx -0.0055556 \approx -0.00556$$

$$\boxed{-0.00556}$$

Solution to Problem 2(f)

$$B_{30} : x = 32 + 1.2 \times 29 = 66.8, \quad y = 2.5 + 0.6 \times 29 = 19.9$$

$$y = -0.005556 \cdot 66.8^2 + 45 \approx -24.798 + 45 \approx 20.202$$

$$19.9 < 20.202 \implies \text{Cannot reach}$$

$$\boxed{\text{Cannot reach}}$$

Alternative Solutions to Problem 2

Alternative Solution to Problem 2(a)(ii)

Use arithmetic sequence: $x_n = 32 + 1.2(n-1)$, $y_n = 2.5 + 0.6(n-1)$.

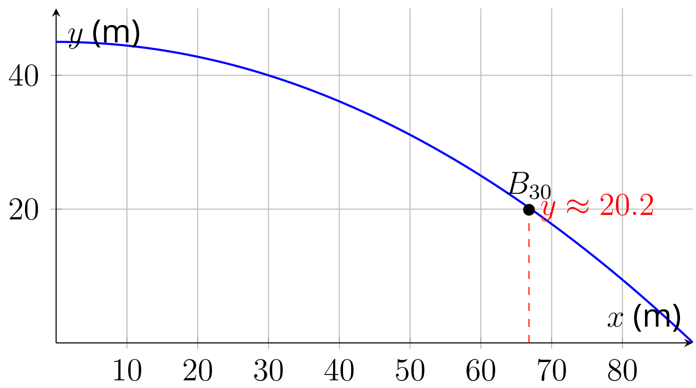
Alternative Solution to Problem 2(d)(ii)

Differentiate $x = 180 \sin\theta \cos\theta$ and set to zero.

Strategy for Projectile Motion

1. **Sequence**: Model seat positions as arithmetic sequences.
2. **Kinematics**: Derive velocity and displacement from acceleration.
3. **Parabola**: Fit boundary using vertex and range.
4. **Comparison**: Check seat coordinates against boundary.

Visualization



Explanation: Parabolic boundary with B_{30} below the maximum height at $x = 66.8$.

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Gift Launcher:

- **(a)(i)**: **A1 A1** for x, y coordinates of B_5.
- **(a)(ii)**: **M1 A1 A1** for sequence method, x, y expressions.
- **(b)(i)**: **M1 A1 A1** for integration, v_x, v_y.
- **(b)(ii)**: **A1 M1 A1** for vertical launch, equation, time.
- **(c)**: **M1 A1** for substituting t, computing height.
- **(d)(i)**: **M1 A1 A1** for time to ground, substitution, expression.
- **(d)(ii)**: **M1 A1** for maximizing $\sin 2\theta$, value.
- **(e)(i)**: **A1** for c from maximum height.
- **(e)(ii)**: **M1 A1** for vertex derivative, b.
- **(e)(iii)**: **M1 A1 A1** for using range point, solving, a.
- **(f)**: **M1 A1 M1 A1 R1** for computing B_{30}, coordinates, parabola height, comparison, conclusion.

Total [29 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Incorrect sequence	Wrong increments for B_n in 2(a)(ii).	Verify $\Delta x = 1.2$, $\Delta y = 0.6$.
Velocity error	Misintegrating acceleration in 2(b)(i).	Check integration constants.
Parabola mistake	Incorrect a calculation in 2(e)(iii).	Use correct range ($x = 90$, $y = 0$).
Reachability error	Wrong B_{30} coordinates in 2(f).	Recalculate sequence for $n = 30$.

Key Takeaways

- Arithmetic sequences model regular spatial patterns.
- Projectile motion follows kinematic equations.
- Parabolic boundaries define reachable regions.
- Coordinate comparisons determine accessibility.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC to compute projectile trajectories.
- **IB Tip**: Sketch seat layout and parabola to visualize.
- **Shortcut**: Store sequence formulas in GDC.
- **Verification**: Confirm parabola vertex and range points.

Basic Foundational Theory

- **Arithmetic Sequence**: $a_n = a_1 + (n-1)d$.
- **Projectile Motion**: $y = v_{0y}t - \frac{1}{2}gt^2$, $x = v_{0x}t$.
- **Parabola**: $y = ax^2 + bx + c$.
- **Kinematics**: $v = u + at$.

Practice Problems

Practice Problem 1: Chi-Squared Test

Perform a chi-squared test on a 2x3 contingency table. [2 marks]

Solution to Practice Problem 1

Compute expected frequencies and p-value using GDC.

$$\boxed{p\text{-value}}$$

Practice Problem 2: Projectile Motion

Calculate the range of a projectile with given velocity. [2 marks]

Solution to Practice Problem 2

Use $x = \frac{v_0^2 \sin 2\theta}{g}$.

$$\boxed{\text{Range}}$$

Further Problems

Further Problem 1: Binomial Test

Test a proportion with a small sample. [3 marks]

Solution to Further Problem 1

Use exact binomial probabilities.

$$\boxed{\text{Conclusion}}$$

Further Problem 2: Parabolic Trajectory

Determine if a point lies within a projectile's reach. [3 marks]

Solution to Further Problem 2

Compare point to parabolic boundary.

$$\boxed{\text{Reachability}}$$

Challenging Problems

Challenging Problem 1: Welch's t-test

Compare means with unequal variances. [3 marks]

Solution to Challenging Problem 1

Use Welch's t-test formula.

$\boxed{\text{Conclusion}}$

Challenging Problem 2: Non-Standard Projectile

Model projectile with variable acceleration. [3 marks]

Solution to Challenging Problem 2

Solve differential equations numerically.

$\boxed{\text{Trajectory}}$

Conclusion: Your Path to Mathematical Mastery

This guide has provided you with a powerful toolset for tackling IB Math AI HL Paper 3 challenges. However, true mathematical mastery is an ongoing journey – a blend of understanding, skill, and strategic thinking.

Key Takeaways for Exam Success:

- **Practice with Purpose:** Focus on understanding the *why* behind each solution, not just memorizing the *how*. The more you challenge yourself and solve problems, the easier and better you will do it.
- **Embrace Your Mistakes:** Every mistake is an opportunity to learn. Analyze what worked and what you can improve next time.
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