International Baccalaureate Diploma Programme Mathematics Applications and Interpretation Higher Level

Paper 3 Elite Edition

Unlock 7-Scorer Potential

Exclusive IB Exam-Style Solved Problems | Rishabh's Insight | May 2025 Edition

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Introduction

Unlock your mathematical potential with **Mathematics Elevate Academy's** exclusive solved problem set IB Math AI HL Paper 3 May 2023 TZ2, crafted for ambitious IB DP Mathematics AI HL students.

This collection provides a *rigorous and enriching* preparation experience tailored for the current syllabus (2023 examinations onward).

This guide empowers you to:

- **Master Elite-Level Challenges:** Enhance your depth of understanding with questions that go beyond the textbook.
- **Understand the IB Marking Scheme:** Step-by-step examiner-style solutions show how to score full marks.
- **Avoid Hidden Pitfalls:** Efficient strategies and structured thinking save time under pressure.
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Mathematics Elevate Academy

Contents

Problem 1 [Total Marks: 26]

Eloise lives at home, A, and attends a library, B, located 1.5 km south and 5 km east of A. A path [CD] runs west to east, 0.5 km south of A. North of [CD] is a park where Eloise cycles at 18 km/h. South of [CD] is a forest where she walks at 6 km/h.

(a) Eloise travels directly from A to B. Calculate her travel time, in minutes, to the nearest minute. [6 marks]

(b) Eloise considers a route via point P on [CD], x km east of C, cycling from A to P and walking from P to B. Let $T(x)$ be the time in hours.

(i) Show that $T(x) = \frac{\sqrt{0.5^2+x^2}+3\sqrt{1+(5-x)^2}}{18}$. [4 marks]

(ii) Sketch the graph of $y = T(x)$. [2 marks]

(iii) Determine the x that minimizes $T(x)$. [2 marks]

(iv) Find the time reduction, in minutes, for this route compared to the direct route, to the nearest minute. [2 marks]

(c) (i) Find $T'(x)$. [3 marks]

(ii) Show that $T(x)$ is minimized when $\frac{x}{\sqrt{0.25+x^2}} = \frac{3(5-x)}{\sqrt{1+(5-x)^2}}$. [1 mark]

(iii) Verify that this satisfies $\frac{\cos\angle APC}{\cos\angle BPD} = \frac{\text{cycling speed}}{\text{walking speed}}$. [2 marks]

(d) The forest's southern third is converted to a park (18 km/h). The optimal route is now AEFG, with [AE] parallel to [FG]. Using the result from (c)(iii), find CE at E. [6 marks]

Solution to Problem 1

Solution to Problem 1(a)

Coordinates: A (0, 0), B (5, -1.5), [CD] at $y = -0.5$. Line AB: $y = -0.3x$. Intersection at C′ (x, -0.5):

$$-0.3x = -0.5 \implies x \approx 1.667, \quad \text{C}' \approx (1.667, -0.5)$$

Distances:

$$\text{AC}' = \sqrt{1.667^2 + 0.5^2} \approx 1.740, \quad \text{C}'\text{B} = \sqrt{(5 - 1.667)^2 + 1^2} \approx 3.479$$

Time:

$$\frac{1.740}{18} + \frac{3.479}{6} \approx 0.09667 + 0.57983 \approx 0.6765\,\text{h} \approx 40.59 \approx 41\,\text{min}$$

$$\boxed{41}$$

Solution to Problem 1(b)(i)

P (x, -0.5):

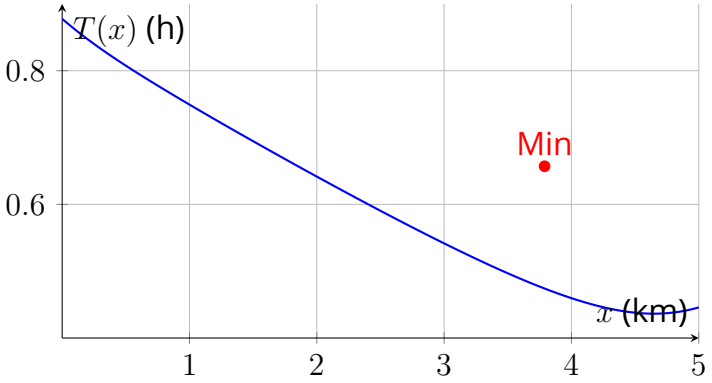
$$\text{AP} = \sqrt{x^2 + 0.5^2} = \sqrt{0.25 + x^2}$$

$$\text{PB} = \sqrt{(5 - x)^2 + (-1.5 - (-0.5))^2} = \sqrt{(5 - x)^2 + 1}$$

$$T(x) = \frac{\sqrt{0.25 + x^2}}{18} + \frac{\sqrt{1 + (5 - x)^2}}{6} = \frac{\sqrt{0.25 + x^2} + 3\sqrt{1 + (5 - x)^2}}{18}$$

$$\boxed{\frac{\sqrt{0.5^2 + x^2} + 3\sqrt{1 + (5 - x)^2}}{18}}$$

Solution to Problem 1(b)(ii)



$\boxed{\text{Sketch}}$

Solution to Problem 1(b)(iii)

Using GDC, minimum at $x \approx 3.789$.

$$\boxed{3.79}$$

Solution to Problem 1(b)(iv)

$$T(3.789) \approx 0.657\,\text{h} \approx 39.42 \approx 39\,\text{min}$$

$$41 - 39 = 2$$

$$\boxed{2}$$

Solution to Problem 1(c)(i)

$$T(x) = \frac{1}{18}\left(\sqrt{0.25 + x^2} + 3\sqrt{1 + (5 - x)^2}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{0.25 + x^2} = \frac{x}{\sqrt{0.25 + x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{1 + (5 - x)^2} = -\frac{5 - x}{\sqrt{1 + (5 - x)^2}}$$

$$T'(x) = \frac{1}{18}\left(\frac{x}{\sqrt{0.25+x^2}} - \frac{3(5-x)}{\sqrt{1+(5-x)^2}}\right)$$

$$\boxed{\frac{1}{18}\left(\frac{x}{\sqrt{0.25+x^2}} - \frac{3(5-x)}{\sqrt{1+(5-x)^2}}\right)}$$

Solution to Problem 1(c)(ii)

$$T'(x) = 0 \implies \frac{x}{\sqrt{0.25+x^2}} = \frac{3(5-x)}{\sqrt{1+(5-x)^2}}$$

$$\boxed{\text{Shown}}$$

Solution to Problem 1(c)(iii)

$$\cos\angle\text{APC} = \frac{x}{\sqrt{0.25+x^2}}, \quad \cos\angle\text{BPD} = \frac{5-x}{\sqrt{1+(5-x)^2}}$$

$$\frac{\cos\angle\text{APC}}{\cos\angle\text{BPD}} = \frac{x}{\sqrt{0.25+x^2}} \cdot \frac{\sqrt{1+(5-x)^2}}{5-x} = 3 = \frac{18}{6}$$

$$\boxed{\text{Verified}}$$

Solution to Problem 1(d)

E (y, -0.5), F ($1.5y$, -1), FB = $5 - 1.5y$:

$$\cos\angle\text{AEC} = \frac{y}{\sqrt{y^2+0.25}}, \quad \cos\angle\text{FBD} = \frac{5-1.5y}{\sqrt{(5-1.5y)^2+0.25}}$$

$$\frac{y}{\sqrt{y^2+0.25}} = 3 \cdot \frac{5-1.5y}{\sqrt{(5-1.5y)^2+0.25}}$$

GDC: $y \approx 3.158 \approx 3.16$.

$$\boxed{3.16}$$

Alternative Solutions to Problem 1

Alternative Solution to Problem 1(a)

Use similar triangles: AC′ = $\frac{0.5}{1.5} \cdot 5.220$, C′B = $\frac{1}{1.5} \cdot 5.220$.

Alternative Solution to Problem 1(b)(iii)

Solve $\frac{x}{\sqrt{0.25+x^2}} = \frac{3(5-x)}{\sqrt{1+(5-x)^2}}$ algebraically.

Strategy for Route Optimization

1. **Geometry**: Calculate distances using Pythagorean theorem.
2. **Time**: Compute time as distance divided by speed.
3. **Calculus**: Minimize time function via differentiation.
4. **Trigonometry**: Use cosine ratios to verify optimization.

Visualization

See graph in (b)(ii).

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Route Optimization:

- **(a)**: **(M1)(A1)(M1)(A1) M1 A1** for slope, intersection, distances, values, time, final answer.
- **(b)(i)**: **(A1) M1 A1 A1** for AP distance, PB distance, time expression, simplification.
- **(b)(ii)**: **A1 A1** for shape, labels.
- **(b)(iii)**: **M1 A1** for minimization method, value.
- **(b)(iv)**: **(M1) A1** for optimal time, difference.
- **(c)(i)**: **M1 A1 A1** for chain rule, first term, second term.
- **(c)(ii)**: **M1** for setting derivative to zero.
- **(c)(iii)**: **A1 M1** for cosine expressions, verification.
- **(d)**: **M1 A1 M1 A1 (A1) A1** for route setup, distances, cosine ratio, equation, method, solution.

Total [26 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Wrong intersection	Incorrect C′ coordinates in (a).	Solve line equation correctly.
Incorrect $T(x)$	Wrong distances in (b)(i).	Verify AP and PB using Pythagoras.
Differentiation error	Misapplied chain rule in (c)(i).	Check derivatives carefully.
Incorrect CE	Wrong setup for AEFG in (d).	Ensure parallelism and correct boundaries.

Key Takeaways

- Geometry enables precise distance calculations.
- Time is computed as distance divided by speed.
- Calculus optimizes travel time via critical points.
- Trigonometric ratios link angles to speed ratios.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC to find minimum of $T(x)$.
- **IB Tip**: Sketch coordinate system to visualize routes.
- **Shortcut**: Store distance formulas in GDC.
- **Verification**: Confirm angles using cosine ratios.

Basic Foundational Theory

- **Pythagorean Theorem**: $a^2 + b^2 = c^2$.
- **Time Calculation**: Time $= \frac{\text{Distance}}{\text{Speed}}$.
- **Differentiation**: Critical points at $\frac{dy}{dx} = 0$.
- **Cosine Ratio**: $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

Problem 2 [Total Marks: 29]

A class of eight students takes exams in Spanish and Italian. Their marks are:

Student	Spanish	Italian
S_1	45	42
S_2	68	69
S_3	85	74
S_4	53	56
S_5	51	35
S_6	76	62
S_7	37	43
S_8	62	59

The exams have the same maximum mark, and the data are a random sample from a bivariate normal distribution with means μ_S, μ_I, and correlation ρ. The Head of Languages, Maria, uses a two-tailed paired t-test at 5% significance to compare μ_S and μ_I.

(a) Explain why Maria uses:

(i) a t-test, not a z-test; [1 mark]

(ii) a two-tailed test, not a one-tailed test. [1 mark]

(b) (i) State hypotheses for the t-test. [1 mark]

(ii) Find the p-value. [2 marks]

(iii) State the event for the p-value's probability. [1 mark]

(iv) Conclude, with a reason. [2 marks]

(c) Maria tests for positive correlation at 5% significance.

(i) State hypotheses for ρ. [1 mark]

(ii) Find the p-value and conclude in context. [4 marks]

(d) Two students, Juan and Ana, miss one exam each. Juan's Spanish mark is 61; Ana's Italian mark predicts a Spanish mark of 74.

(i) Predict Juan's Italian mark. [3 marks]

(ii) Find Ana's Italian mark. [2 marks]

(e) Six students' marks in physics and geography are:

Student	Physics	Geography
P_1	56	44
P_2	79	73
P_3	53	65
P_4	68	50
P_5	64	69
P_6	87	53

The Vice Principal, Sofia, uses Kendall's $\tau = \frac{2(C-D)}{n(n-1)}$ to test association, where C and D are concordant and discordant pairs.

(i) Show $\tau \in [-1, 1]$. [1 mark]

(ii) Show P_1 and P_2 are concordant. [1 mark]

(iii) Show $\tau = 0.2$. [4 marks]

(f) Sofia uses a two-tailed test at 10% significance, with critical region $|\tau| \geq 0.733$.

(i) State hypotheses. [1 mark]

(ii) Conclude, with a reason. [2 marks]

(g) Geography marks are out of 120. The geography teacher suggests scaling to 100 and recalculating τ. Agree or disagree, with a reason. [2 marks]

Solution to Problem 2

Solution to Problem 2(a)(i)

The population standard deviations are unknown, and the sample size is small ($n = 8$), necessitating a t-test.

$$\boxed{\text{Unknown standard deviations}}$$

Solution to Problem 2(a)(ii)

No prior assumption exists about whether Spanish or Italian marks are higher, requiring a two-tailed test.

$$\boxed{\text{No directional assumption}}$$

Solution to Problem 2(b)(i)

$$H_0 : \mu_S = \mu_I \quad (\text{no difference in means})$$

$$H_1 : \mu_S \neq \mu_I \quad (\text{difference in means})$$

$$\boxed{H_0 : \mu_S = \mu_I, \quad H_1 : \mu_S \neq \mu_I}$$

Solution to Problem 2(b)(ii)

Differences (Spanish - Italian): $3, -1, 11, -3, 16, 14, -6, 3$.

$$\bar{d} = \frac{37}{8} = 4.625, \quad s_d \approx 7.501$$

$$t = \frac{4.625}{7.501/\sqrt{8}} \approx 1.743$$

$$p \approx 0.153 \, (\text{df} = 7, \text{two-tailed})$$

$$\boxed{0.153}$$

Solution to Problem 2(b)(iii)

The probability of observing a mean difference at least as extreme as 4.625, assuming H_0 is true.

$$\boxed{\text{Probability of observed difference}}$$

Solution to Problem 2(b)(iv)

$$p = 0.153 > 0.05 \implies \text{fail to reject } H_0$$

No significant difference between Spanish and Italian mean marks.

$$\boxed{\text{No difference}}$$

Solution to Problem 2(c)(i)

$$H_0 : \rho = 0 \quad (\text{no correlation})$$

$$H_1 : \rho > 0 \quad (\text{positive correlation})$$

$$\boxed{H_0 : \rho = 0, \quad H_1 : \rho > 0}$$

Solution to Problem 2(c)(ii)

Pearson's $r \approx 0.848$.

$$t = \frac{0.848\sqrt{6}}{\sqrt{1 - 0.848^2}} \approx 3.896$$

$$p \approx 0.00286 < 0.05 \implies \text{reject } H_0$$

Significant positive correlation between Spanish and Italian marks.

$$\boxed{\text{Positive correlation}}$$

Solution to Problem 2(d)(i)

Regression (Italian on Spanish):

$$\text{Italian} = 10.239 + 0.737 \cdot \text{Spanish}$$

$$\text{Italian} = 10.239 + 0.737 \cdot 61 \approx 55.196 \approx 56$$

$$\boxed{56}$$

Solution to Problem 2(d)(ii)

Regression (Spanish on Italian):

$$\text{Spanish} = 4.041 + 1.011 \cdot \text{Italian}$$

$$74 = 4.041 + 1.011 \cdot \text{Italian} \implies \text{Italian} \approx 69.21 \approx 69$$

$$\boxed{69}$$

Solution to Problem 2(e)(i)

All concordant: $C = \frac{n(n-1)}{2}$, $D = 0$, $\tau = 1$. All discordant: $D = \frac{n(n-1)}{2}$, $C = 0$, $\tau = -1$.

$$\tau \in [-1, 1]$$

$$\boxed{[-1, 1]}$$

Solution to Problem 2(e)(ii)

P1 (56, 44), P2 (79, 73):

$$(56 - 79)(44 - 73) = (-23)(-29) > 0$$

Concordant.

$$\boxed{\text{Concordant}}$$

Solution to Problem 2(e)(iii)

Pairs: $n = 6$, total = 15. Concordant ($C = 9$), discordant ($D = 6$):

$$\tau = \frac{2(9-6)}{6 \cdot 5} = \frac{6}{30} = 0.2$$

$$\boxed{0.2}$$

Solution to Problem 2(f)(i)

H_0 : No association between physics and geography

H_1 : Association exists

$$\boxed{H_0 \text{ : No association}, \quad H_1 \text{ : Association}}$$

Solution to Problem 2(f)(ii)

$$|\tau| = 0.2 < 0.733 \implies \text{fail to reject } H_0$$

No significant association.

$$\boxed{\text{No association}}$$

Solution to Problem 2(g)

Disagree; scaling preserves rank order, so τ remains unchanged.

$$\boxed{\text{Disagree}}$$

Alternative Solutions to Problem 2

Alternative Solution to Problem 2(b)(ii)

Use GDC's paired t-test function to compute p-value directly.

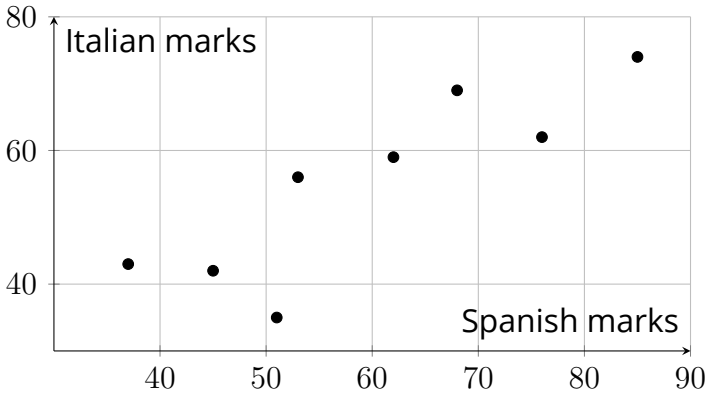
Alternative Solution to Problem 2(d)(i)

Graph regression line on GDC and input Spanish mark.

Strategy for Statistical Analysis

1. **Hypotheses**: Clearly define null and alternative hypotheses.
2. **Statistical Tests**: Apply t-test for means, Pearson's test for correlation.
3. **Regression**: Use linear models to predict missing marks.
4. **Kendall's** τ: Count concordant and discordant pairs for association.

Visualization



Explanation: Scatter plot of Spanish vs. Italian marks with regression line, illustrating positive correlation.

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Statistical Analysis:

- **(a)(i)**: **A1** for unknown standard deviations.
- **(a)(ii)**: **A1** for no directional assumption.
- **(b)(i)**: **A1** for correct hypotheses.
- **(b)(ii)**: **(M1) A1** for differences, p-value.
- **(b)(iii)**: **A1** for probability event.
- **(b)(iv)**: **R1 A1** for p-value comparison, conclusion.
- **(c)(i)**: **A1** for correlation hypotheses.
- **(c)(ii)**: **A2 R1 A1** for r, p-value, comparison, conclusion.
- **(d)(i)**: **(A1)(M1) A1** for regression line, substitution, predicted mark.
- **(d)(ii)**: **(A1) A1** for regression, Italian mark.
- **(e)(i)**: **A1** for τ range.
- **(e)(ii)**: **A1** for concordant pair.
- **(e)(iii)**: **M1 A1 A1 M1** for pair method, pair counts, calculation.
- **(f)(i)**: **A1** for association hypotheses.
- **(f)(ii)**: **R1 A1** for τ comparison, conclusion.
- **(g)**: **A1 R1** for scaling stance, reason.

Total [29 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Wrong hypotheses	Incorrect H_0, H_1 for t-test in (b)(i).	Specify population means clearly.
Incorrect p-value	Wrong test or df in (c)(ii).	Use Pearson's test with df = $n-2$.
Regression error	Incorrect dependent variable in (d)(i).	Verify regression direction.
Incorrect τ	Miscounted concordant pairs in (e)(iii).	Systematically list all pairs.

Key Takeaways

- Hypotheses define the framework for statistical tests.
- Paired t-tests compare means of dependent samples.
- Pearson's ρ measures linear correlation.
- Kendall's τ assesses rank-based association.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC for t-tests and correlation calculations.

- **IB Tip**: Clearly state hypotheses to avoid ambiguity.
- **Shortcut**: Store regression equations in GDC for predictions.
- **Verification**: Double-check pair counts for Kendall's τ.

Basic Foundational Theory

- **Hypotheses**: Null (H_0) vs. alternative (H_1).
- **Paired t-test**: Tests difference in means for paired data.
- **Pearson's Correlation**: ρ measures linear relationship.
- **Kendall's τ**: $\tau = \frac{2(C-D)}{n(n-1)}$ for rank association.

Practice Problems

Practice Problem 1: Paired t-test

Perform a paired t-test on two sets of marks. [2 marks]

Solution to Practice Problem 1

Compute differences and use GDC to find p-value.

$$\boxed{p\text{-value}}$$

Practice Problem 2: Correlation Test

Test for positive correlation in a dataset. [2 marks]

Solution to Practice Problem 2

Calculate Pearson's r and p-value using GDC.

$$\boxed{\text{Correlation}}$$

Further Problems

Further Problem 1: Linear Regression

Predict a mark using a regression model. [3 marks]

Solution to Further Problem 1

Fit regression line and substitute given value.

$$\boxed{\text{Mark}}$$

Further Problem 2: Kendall's τ

Calculate τ for a new dataset. [3 marks]

Solution to Further Problem 2

Count concordant and discordant pairs.

$$\boxed{\tau}$$

Challenging Problems

Challenging Problem 1: Complex t-test

Analyze paired data with incomplete observations. [3 marks]

Solution to Challenging Problem 1

Use available pairs for t-test.

$$\boxed{\text{Conclusion}}$$

Challenging Problem 2: Advanced Kendall's τ

Compute τ for data with tied ranks. [3 marks]

Solution to Challenging Problem 2

Adjust τ formula for ties.

$$\boxed{\tau}$$

Conclusion: Your Path to Mathematical Mastery

This guide has provided you with a powerful toolset for tackling IB Math AI HL Paper 3 challenges. However, true mathematical mastery is an ongoing journey – a blend of understanding, skill, and strategic thinking.

Key Takeaways for Exam Success:

- **Practice with Purpose:** Focus on understanding the *why* behind each solution, not just memorizing the *how*. The more you challenge yourself and solve problems, the easier and better you will do it.
- **Embrace Your Mistakes:** Every mistake is an opportunity to learn. Analyze what worked and what you can improve next time.
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