International Baccalaureate Diploma Programme Mathematics Applications and Interpretation Higher Level

Paper 3 Elite Edition

Unlock 7-Scorer Potential

Exclusive IB Exam-Style Solved Problems | Rishabh's Insight | May 2025 Edition

Mathematics Elevate Academy

Excellence in Further Math Education

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5+ Years of Teaching Experience

Disclaimer

Introduction

Unlock your mathematical potential with **Mathematics Elevate Academy's** exclusive solved problem set IB Math AI HL Paper 3 May 2023 TZ1, crafted for ambitious IB DP Mathematics AI HL students.

This collection provides a *rigorous and enriching* preparation experience tailored for the current syllabus (2023 examinations onward).

This guide empowers you to:

- **Master Elite-Level Challenges:** Enhance your depth of understanding with questions that go beyond the textbook.
- **Understand the IB Marking Scheme:** Step-by-step examiner-style solutions show how to score full marks.
- **Avoid Hidden Pitfalls:** Efficient strategies and structured thinking save time under pressure.
- **Build a Mathematical Toolkit:** Strengthen your command over high-level problem-solving techniques.

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Contents

Problem 1 4
- Solution to Problem 1 . . . 6
- Alternative Solutions to Problem 1 . . . 9
- Visualization . . . 9
- Plots/Graphs . . . 10
- Marking Criteria . . . 10
- Error Analysis . . . 11
- Key Takeaways . . . 11
- Rishabh's Insights . . . 11
- Basic Foundational Theory . . . 12

Problem 2 13
- Solution to Problem 2 . . . 15
- Alternative Solutions to Problem 2 . . . 18
- Visualization . . . 18
- Plots/Graphs . . . 19
- Marking Criteria . . . 19
- Error Analysis . . . 20
- Key Takeaways . . . 20
- Rishabh's Insights . . . 20
- Basic Foundational Theory . . . 21

Practice Problems 22

Further Problems 23

Challenging Problems 24

Conclusion 25

Problem 1 [Total Marks: 26]

In a historical method from 18th-century France, the cost of an oil drum was determined by inserting a rod through a side hole, measuring the length RD (in metres), denoted d. A longer RD implied a higher cost, C, in livres (local currency). When RD was 0.6 m, the cost was 0.90 livres.

(a) Given that C is proportional to d, find an equation for C in terms of d. [3 marks]

(b) For a drum costing 1.08 livres, show that $d = 0.72$. [1 mark]

(c) A mathematician modeled the drum as a cylinder, with R at the midpoint of one side, length $2h$ m, and radius r m. Find r^2 in terms of d and h. [3 marks]

(d) Let the drum's volume be V m^3. Show that $V = \frac{\pi}{2}(d^2h - h^3)$. [2 marks]

(e) For $d = 0.72$:

  (i) Find the volume when $h = 0.48$. [2 marks]

  (ii) Use differentiation to show that $h = \sqrt{0.1728}$ when $\frac{\mathrm{d}V}{\mathrm{d}h} = 0$. [3 marks]

  (iii) Given this h maximizes volume, find the maximum volume. [2 marks]

(f) The mathematician considered a non-cylindrical drum with circular bases of radius 0.24 m and length 0.96 m. The curved surface is formed by rotating a quadratic curve, PRQ, with equation $y = ax^2 + bx + c$, $0 \leq x \leq 0.96$, about the x-axis. R, the vertex, has RD = 0.72. Points P and Q are at $(0, 0.24)$ and $(0.96, 0.24)$. Find the equation of PRQ. [5 marks]

(g) Show that this drum's volume exceeds the maximum volume of any cylindrical drum with $d = 0.72$. [5 marks]

(h) State one assumption, not already given, for these models. [1 mark]

Solution to Problem 1

Solution to Problem 1(a)

Since $C \propto d$, let $C = kd$. Given $C = 0.90$ when $d = 0.6$:

$$0.90 = k \cdot 0.6 \implies k = \frac{0.90}{0.6} = 1.5$$

Thus:

$$C = 1.5d$$

$$\boxed{C = 1.5d}$$

Solution to Problem 1(b)

Using $C = 1.5d$:

$$1.08 = 1.5d \implies d = \frac{1.08}{1.5} = 0.72$$

$$\boxed{0.72}$$

Solution to Problem 1(c)

In the cylinder, RD forms a right triangle with legs h and $2r$:

$$d^2 = h^2 + (2r)^2 = h^2 + 4r^2$$

$$r^2 = \frac{d^2 - h^2}{4}$$

$$\boxed{\frac{d^2 - h^2}{4}}$$

Solution to Problem 1(d)

Cylinder volume:

$$V = \pi r^2 \cdot 2h = \pi \cdot \frac{d^2 - h^2}{4} \cdot 2h = \frac{\pi}{2}(d^2h - h^3)$$

$$\boxed{\frac{\pi}{2}(d^2h - h^3)}$$

Solution to Problem 1(e)(i)

For $d = 0.72$, $h = 0.48$:

$$d^2 = 0.5184, \quad h^3 = 0.48^3 = 0.110592$$

$$V = \frac{\pi}{2}(0.5184 \cdot 0.48 - 0.110592) \approx \frac{\pi}{2} \cdot 0.13824 \approx 0.217104 \approx 0.181$$

$$\boxed{0.181}$$

Solution to Problem 1(e)(ii)

$$V = \frac{\pi}{2}(0.5184h - h^3), \quad \frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\pi}{2}(0.5184 - 3h^2)$$

Set $\frac{\mathrm{d}V}{\mathrm{d}h} = 0$:

$$0.5184 - 3h^2 = 0 \implies h^2 = \frac{0.5184}{3} = 0.1728 \implies h = \sqrt{0.1728}$$

$$\boxed{\sqrt{0.1728}}$$

Solution to Problem 1(e)(iii)

$$h = \sqrt{0.1728} \approx 0.4157, \quad h^3 \approx 0.07179$$

$$V = \frac{\pi}{2}(0.5184 \cdot 0.4157 - 0.07179) \approx \frac{\pi}{2} \cdot 0.14371 \approx 0.2257 \approx 0.187$$

$$\boxed{0.187}$$

Solution to Problem 1(f)

Vertex at R $(0.48, 0.72)$:

$$y = a(x - 0.48)^2 + 0.72$$

At P $(0, 0.24)$:

$$0.24 = a(0 - 0.48)^2 + 0.72 \implies a \cdot 0.2304 = -0.48 \implies a \approx -2.08333$$

$$y = -2.08333(x - 0.48)^2 + 0.72$$

$$\boxed{y = -2.08333(x - 0.48)^2 + 0.72}$$

Solution to Problem 1(g)

$$V = \pi \int_0^{0.96} [-2.08333(x - 0.48)^2 + 0.72]^2 \, dx \approx 0.194$$

$$0.194 > 0.187$$

$$\boxed{\text{Greater volume}}$$

Solution to Problem 1(h)

The drum is full when sold.

$$\boxed{\text{Full drum}}$$

Alternative Solutions to Problem 1

Alternative Solution to Problem 1(e)(ii)

Graph V on GDC to find maximum at $h \approx 0.4157$.

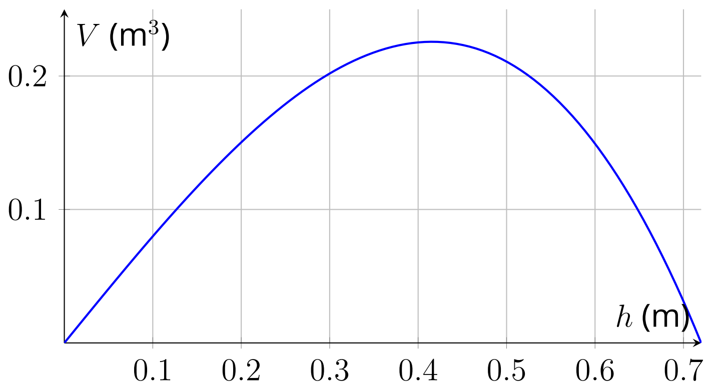
Alternative Solution to Problem 1(g)

Use GDC numerical integration for $\pi \int_0^{0.96} y^2\, dx$.

Strategy for Volume Optimization

1. **Proportionality**: Establish $C = kd$ using given data.
2. **Geometry**: Apply Pythagorean theorem for radius.
3. **Calculus**: Differentiate volume to find maximum.
4. **Integration**: Compute volume of revolution for non-cylindrical drum.

Visualization



Explanation: Plot of volume V vs. half-length h for $d = 0.72$, showing maximum at $h \approx 0.4157$.

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Oil Drum:

- **(a)**: **(M1) A1 A1** for proportionality, constant, equation.
- **(b)**: **M1** for substitution.
- **(c)**: **(M1) A1 A1** for Pythagoras, expression, simplification.
- **(d)**: **M1 A1** for substitution, formula.
- **(e)(i)**: **(M1) A1** for substitution, volume.
- **(e)(ii)**: **M1 A1 A1** for differentiation, equation, solution.
- **(e)(iii)**: **(M1) A1** for substitution, volume.
- **(f)**: **A1 (M1) A1 (M1) A1** for vertex, form, substitution, solving a, equation.
- **(g)**: **(M1) A1 (M1) A1 R1** for integral setup, integrand, integration, volume, comparison.
- **(h)**: **A1** for assumption.

Total [26 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Wrong k	Miscalculated proportionality constant in (a).	Verify with given values.
Incorrect r^2	Wrong triangle setup in (c).	Ensure legs are h and $2r$.
Differentiation error	Incorrect derivative in (e)(ii).	Apply chain rule correctly.
Integration error	Incorrect limits or integrand in (g).	Use x from 0 to 0.96.

Key Takeaways

- Proportionality relates cost to rod length.
- Geometric constraints define cylinder dimensions.
- Calculus optimizes volume via critical points.
- Integration computes volumes of complex shapes.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC for numerical integration.
- **IB Tip**: Simplify expressions before substitution.
- **Shortcut**: Store constants in GDC memory.
- **Verification**: Graph volume function to confirm maximum.

Basic Foundational Theory

- **Proportionality**: $y = kx$.
- **Pythagoras**: $a^2 + b^2 = c^2$.
- **Cylinder Volume**: $V = \pi r^2 h$.
- **Differentiation**: Critical points at $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.
- **Volume of Revolution**: $V = \pi \int_a^b y^2 \, dx$.

Problem 2 [Total Marks: 29]

At an airport, the control tower's base is the origin. Coordinates (x, y, z) represent an aircraft's position (km east, north, and above the tower). At 14:00, aircraft X and Y are at S(120, -98, 12.1) and T(250, -230, 12.1).

(a) Find the distance between X and Y at 14:00. [2 marks]

(b) Both aircraft fly the same path with velocity $\begin{pmatrix} -720 \\ 720 \\ 0 \end{pmatrix}$ km/h. Find their speed. [2 marks]

(c) Regulations require a 12-minute gap on the same path. Find the time for Y to reach S from T and state if they are in conflict. [3 marks]

(d) Write the position vector of X, t hours after 14:00. [1 mark]

(e) Aircraft Z, on a different path, has position $\begin{pmatrix} -480 \\ -49 \\ 10.3 \end{pmatrix} + t \begin{pmatrix} -160 \\ 680 \\ 2.4 \end{pmatrix}$. Regulations require: if vertical distance is less than 360 m, horizontal distance must exceed 12 km.

(i) Find when the distance between X and Z is 12 km. [5 marks]

(ii) Determine if X and Z break regulations. Justify. [5 marks]

(f) A new coordinate system at point U, 2.4 km above the tower, defines positions east and north of U. Aircraft W, at 2.4 km height, has position $\begin{pmatrix} 7.2\cos(42t) \\ 7.2\sin(42t) \end{pmatrix}$. Describe W's path. [3 marks]

(g) Aircraft V, at 2.4 km, follows $\overrightarrow{UV} = \begin{pmatrix} 24 \\ 12 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, with $\boldsymbol{b} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(i) Find $\overrightarrow{\mathrm{UV}} \cdot \boldsymbol{b}$ in terms of λ. [2 marks]

(ii) Find λ for minimum distance from U to the line. [2 marks]

(iii) Find this minimum distance. [2 marks]

(iv) Show that W and V do not break regulations. [2 marks]

Solution to Problem 2

Solution to Problem 2(a)

$$\vec{\mathsf{ST}} = \begin{pmatrix} 250 - 120 \\ -230 - (-98) \\ 12.1 - 12.1 \end{pmatrix} = \begin{pmatrix} 130 \\ -132 \\ 0 \end{pmatrix}$$

$$|\vec{\mathsf{ST}}| = \sqrt{130^2 + 132^2} \approx \sqrt{34324} \approx 185.26 \approx 185$$

$$\boxed{185}$$

Solution to Problem 2(b)

$$\text{Speed} = \sqrt{(-720)^2 + 720^2} = \sqrt{1036800} \approx 1018.04 \approx 1018$$

$$\boxed{1018}$$

Solution to Problem 2(c)

$$\text{Time} = \frac{185}{1018} \approx 0.181728\,\text{h} \approx 10.91\,\text{min}$$

$$10.91 < 12, \quad \text{in conflict}$$

$$\boxed{\text{In conflict}}$$

Solution to Problem 2(d)

$$\boldsymbol{r}_X = \begin{pmatrix} 120 \\ -98 \\ 12.1 \end{pmatrix} + t \begin{pmatrix} -720 \\ 720 \\ 0 \end{pmatrix} = \begin{pmatrix} 120 - 720t \\ -98 + 720t \\ 12.1 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 120 - 720t \\ -98 + 720t \\ 12.1 \end{pmatrix}}$$

Solution to Problem 2(e)(i)

$$\boldsymbol{r}_X - \boldsymbol{r}_Z = \begin{pmatrix} 600 - 560t \\ -49 + 40t \\ 1.8 - 2.4t \end{pmatrix}$$

$$(600 - 560t)^2 + (-49 + 40t)^2 + (1.8 - 2.4t)^2 = 144$$

Solve quadratic (via GDC):

$$t \approx 0.982, 1.022$$

$$\boxed{0.982, 1.022}$$

Solution to Problem 2(e)(ii)

Vertical distance:

$$|1.8 - 2.4t| < 0.36 \implies 0.6 < t < 0.9$$

At $t = 0.75$, horizontal distance:

$$\sqrt{(600 - 560 \cdot 0.75)^2 + (-49 + 40 \cdot 0.75)^2} \approx 180.34 > 12$$

No violation.

$$\boxed{\text{No violation}}$$

Solution to Problem 2(f)

$$x^2 + y^2 = (7.2\cos(42t))^2 + (7.2\sin(42t))^2 = 7.2^2$$

Circular path, radius 7.2 km, centered at U.

$$\boxed{\text{Circle, radius 7.2, centered at U}}$$

Solution to Problem 2(g)(i)

$$\overrightarrow{\mathrm{UV}} = \begin{pmatrix} 24-\lambda \\ 12+\lambda \end{pmatrix}, \quad \overrightarrow{\mathrm{UV}} \cdot \boldsymbol{b} = (24-\lambda)\cdot(-1) + (12+\lambda)\cdot 1 = -12+2\lambda$$

$$\boxed{-12+2\lambda}$$

Solution to Problem 2(g)(ii)

$$-12+2\lambda = 0 \implies \lambda = 6$$

$$\boxed{6}$$

Solution to Problem 2(g)(iii)

At $\lambda = 6$:

$$\overrightarrow{\mathrm{UV}} = \begin{pmatrix} 18 \\ 18 \end{pmatrix}, \quad |\overrightarrow{\mathrm{UV}}| = \sqrt{18^2+18^2} \approx 25.455 \approx 25.5$$

$$\boxed{25.5}$$

Solution to Problem 2(g)(iv)

Vertical distance = 0. Minimum horizontal distance:

$$25.5 - 7.2 = 18.3 > 12$$

No violation.

$$\boxed{\text{No violation}}$$

Alternative Solutions to Problem 2

Alternative Solution to Problem 2(e)(i)

Use GDC to solve $\sqrt{(600-560t)^2+(-49+40t)^2+(1.8-2.4t)^2}=12$.

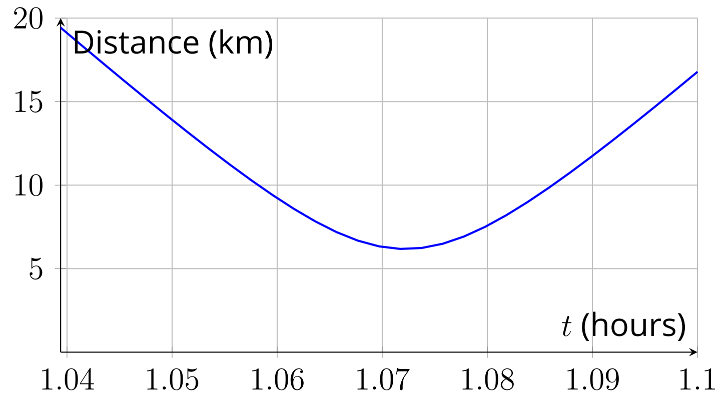
Alternative Solution to Problem 2(e)(ii)

Check vertical distance at $t = 0.982, 1.022$ to confirm they are outside regulation interval.

Strategy for Vector Analysis

1. **Vectors**: Compute distances using vector modulus.
2. **Speed**: Calculate magnitude of velocity vector.
3. **Time**: Use distance over speed for travel time.
4. **Regulations**: Verify vertical and horizontal distance constraints.

Visualization



Explanation: Plot of distance between aircraft X and Z vs. time, showing points where distance equals 12 km.

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Aircraft:

- **(a)**: **(M1) A1** for vector difference, distance.
- **(b)**: **(M1) A1** for velocity modulus, speed.
- **(c)**: **(M1) A1 R1** for time calculation, conversion, comparison.
- **(d)**: **A1** for position vector.
- **(e)(i)**: **(M1)(A1) (M1) A1 A1** for vector difference, expression, modulus, times.
- **(e)(ii)**: **M1 A1 A1 R1 A1** for vertical component, inequality, interval, comparison, conclusion.
- **(f)**: **A1 A1 A1** for shape, radius, center.
- **(g)(i)**: **(M1) A1** for scalar product, expression.
- **(g)(ii)**: **(M1) A1** for perpendicularity, solution.
- **(g)(iii)**: **(M1) A1** for vector, distance.
- **(g)(iv)**: **M1 R1** for comparison, conclusion.

Total [29 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Wrong distance	Incorrect vector components in (a).	Verify coordinates of S and T.
Time error	Incorrect unit conversion in (c).	Convert hours to minutes accurately.
Interval error	Wrong bounds for vertical distance in (e)(ii).	Solve $\|1.8 - 2.4t\| < 0.36$ correctly.
Scalar product	Incorrect vector in (g)(i).	Use direction vector $\boldsymbol{b}$.

Key Takeaways

- Vector geometry computes distances and positions.
- Speed is the magnitude of the velocity vector.
- Time calculations ensure regulation compliance.
- Scalar products find minimum distances to lines.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC to compute vector moduli.

- **IB Tip**: Convert units early to avoid errors.
- **Shortcut**: Store position vectors in GDC.
- **Verification**: Graph distance functions to confirm intersections.

Basic Foundational Theory

- **Vector Distance**: $|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$.
- **Speed**: Magnitude of velocity vector.
- **Scalar Product**: $\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y$.
- **Parametric Equations**: $\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v}$.

Practice Problems

Practice Problem 1: Vector Distance

Calculate the distance between two aircraft at given coordinates. [2 marks]

Solution to Practice Problem 1

Compute the modulus of the difference vector.

$$\boxed{\text{Distance}}$$

Practice Problem 2: Volume Optimization

Maximize the volume of a cylinder with given constraints. [2 marks]

Solution to Practice Problem 2

Differentiate the volume function to find the critical point.

$$\boxed{\text{Maximum}}$$

Further Problems

Further Problem 1: Scalar Product

Find the minimum distance from a point to a line using scalar product. [3 marks]

Solution to Further Problem 1

Set the scalar product to zero to find the perpendicular point.

$$\boxed{\text{Distance}}$$

Further Problem 2: Circular Path

Describe the motion of an aircraft on a circular path. [3 marks]

Solution to Further Problem 2

Identify the radius and center from parametric equations.

$$\boxed{\text{Circle}}$$

Challenging Problems

Challenging Problem 1: Complex Vectors

Analyze the paths of multiple aircraft using vector equations. [3 marks]

Solution to Challenging Problem 1

Use parametric equations to determine intersections.

$$\boxed{\text{Analysis}}$$

Challenging Problem 2: Advanced Optimization

Optimize the volume of a complex drum shape. [3 marks]

Solution to Challenging Problem 2

Apply calculus and integration to maximize volume.

$$\boxed{\text{Volume}}$$

Conclusion: Your Path to Mathematical Mastery

This guide has provided you with a powerful toolset for tackling IB Math AI HL Paper 3 challenges. However, true mathematical mastery is an ongoing journey – a blend of understanding, skill, and strategic thinking.

Key Takeaways for Exam Success:

- **Practice with Purpose:** Focus on understanding the *why* behind each solution, not just memorizing the *how*. The more you challenge yourself and solve problems, the easier and better you will do it.
- **Embrace Your Mistakes:** Every mistake is an opportunity to learn. Analyze what worked and what you can improve next time.
- **Time is a Crucial Asset:** Simulate the exam and prepare well to achieve success.

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