International Baccalaureate Diploma Programme Mathematics Applications and Interpretation Higher Level

Paper 3 Elite Edition

Unlock 7-Scorer Potential

Exclusive IB Exam-Style Solved Problems | Rishabh's Insight | May 2025 Edition

Mathematics Elevate Academy

Excellence in Further Math Education

Rishabh Kumar

Founder, Mathematics Elevate Academy

Math by Rishabh - Elite Private Mentor for IB Math HL

Alumnus of IIT Guwahati & the Indian Statistical Institute

5+ Years of Teaching Experience

Apply For Personalized Mentorship

www.mathematicselevateacademy.com

www.linkedin.com/in/rishabh-kumar-iitg-isi/

Disclaimer

This material contains solutions to problems inspired by the IB Mathematics: Applications and Interpretation Higher Level 2022 Paper 3, Time Zone 2 (TZ2). The original questions have been paraphrased and restructured to support learning and avoid direct reproduction of copyrighted material. This content is intended solely for educational and non-commercial use. The International Baccalaureate Organization (IBO) holds the copyright to the original examination materials and does not endorse or approve this work. All original intellectual property rights remain with the IBO.

Introduction

Unlock your mathematical potential with **Mathematics Elevate Academy's** exclusive solved problem set IB Math AI HL Paper 3 May 2022 TZ2, crafted for ambitious IB DP Mathematics AI HL students.

This collection provides a *rigorous and enriching* preparation experience tailored for the current syllabus (2022 examinations onward).

This guide empowers you to:

- **Master Elite-Level Challenges:** Enhance your depth of understanding with questions that go beyond the textbook.
- **Understand the IB Marking Scheme:** Step-by-step examiner-style solutions show how to score full marks.
- **Avoid Hidden Pitfalls:** Efficient strategies and structured thinking save time under pressure.
- **Build a Mathematical Toolkit:** Strengthen your command over high-level problem-solving techniques.

Aiming for the Ivy League, Oxbridge, or to advance your math skills?
Join my exclusive mentorship — application-based only. For serious, high-achieving students. Only 6 seats worldwide annually.
Ready to transform your exam performance?

Contents

Problem 1 [Total Marks: 27]

Somsak, a store manager in Chiang Mai, analyzes daily sales of mangoes to optimize inventory. He collects data over 100 days, recording the number of mangoes sold each day:

Number of mangoes sold	0	1	2	3	4	5	6	7	8	9	10
Number of days	2	9	14	13	20	15	12	9	3	1	2

He suspects the data follows a Poisson distribution.

(a) (i) Calculate the mean and variance of the sample data. [2 marks]

(ii) Explain why Somsak believes the data follows a Poisson distribution. [1 mark]

(b) State one assumption required for the sales to follow a Poisson distribution. [1 mark]

(c) Historical records show an average of 4.3 mangoes sold daily. The expected frequencies for a Poisson distribution with mean 4.3 over 100 days are:

Number of mangoes sold	≤ 1	2	3	4	5	6	≥ 7
Expected frequency	p	12.879	18.463	q	15.873	11.374	r

Find p, q, and r to three decimal places. [5 marks]

(d) Somsak performs a χ^2 goodness of fit test at the 5% significance level to test if the data follows a Poisson distribution with mean 4.3.

(i) State the degrees of freedom for the test. [1 mark]

(ii) Conduct the test and state the conclusion with reasoning. [7 marks]

(e) Somsak claims that a 350 THB daily radio advertisement will increase mango sales. The store owner, Niran, doubts it will increase profits. After advertising for 70 days, Somsak records 343 mangoes sold, with a 510 THB profit per mango.

(i) Perform a hypothesis test at the 5% significance level to determine if sales increased compared to historical records. [6 marks]

(ii) State the probability of a Type I error. [1 mark]

(f) Evaluate whether the advertising was beneficial, considering both claims. [3 marks]

Solution to Problem 1

Solution to Problem 1(a)(i)

Calculate mean:

$$\sum(x \cdot f) = 0 \cdot 2 + 1 \cdot 9 + 2 \cdot 14 + 3 \cdot 13 + 4 \cdot 20 + 5 \cdot 15 + 6 \cdot 12 + 7 \cdot 9 + 8 \cdot 3 + 9 \cdot 1 + 10 \cdot 2 = 419$$

$$\text{Mean} = \frac{419}{100} = 4.19$$

Calculate variance:

$$\sum(x^2 \cdot f) = 0^2 \cdot 2 + 1^2 \cdot 9 + \cdots + 10^2 \cdot 2 = 2223$$

$$s^2 = \frac{2223}{100} - 4.19^2 = 22.23 - 17.5561 \approx 4.67$$

$$\boxed{4.19, 4.67}$$

Solution to Problem 1(a)(ii)

The mean (4.19) is close to the variance (4.67), a key property of the Poisson distribution.

$$\boxed{\text{Mean approximately equals variance}}$$

Solution to Problem 1(b)

Sales on different days are independent.

$$\boxed{\text{Independent sales}}$$

Solution to Problem 1(c)

For Poisson $\lambda = 4.3$:

$$p = 100 \cdot (\mathsf{P}(X = 0) + \mathsf{P}(X = 1)) \approx 100 \cdot (0.013569 + 0.058346) = 7.192$$

$$q = 100 \cdot \mathsf{P}(X = 4) = 100 \cdot \frac{4.3^4 e^{-4.3}}{4!} \approx 19.323$$

$$r = 100 \cdot (1 - \mathsf{P}(X \leq 6)) \approx 100 \cdot 0.144275 = 14.428$$

$$\boxed{7.192, 19.323, 14.428}$$

Solution to Problem 1(d)(i)

Categories: 7, parameters: 0, so:

$$\text{df} = 7 - 1 = 6$$

$$\boxed{6}$$

Solution to Problem 1(d)(ii)

Hypotheses:

$$\text{H}_0 : \text{Data follows Poisson with mean 4.3}, \quad \text{H}_1 : \text{Data does not follow Poisson}$$

Observed: {11, 14, 13, 20, 15, 12, 15}, Expected: {7.192, 12.879, 18.463, 19.323, 15.873, 11.374, 14.428}.

$$\chi^2 = \sum \frac{(O - E)^2}{E} \approx 12.531$$

Critical value (df $= 6, \alpha = 0.05$): 12.592. Since $12.531 < 12.592$, do not reject H_0.

$$\boxed{\text{Follows Poisson}}$$

Solution to Problem 1(e)(i)

Hypotheses:

$$\mathrm{H}_0 : \mu = 4.3 \cdot 70 = 301, \quad \mathrm{H}_1 : \mu > 301$$

Using normal approximation, critical value: $301 + 1.645 \cdot \sqrt{301} \approx 329.54$. Observed 343 > 329.54, reject H_0.

$$\boxed{\text{Sales increased}}$$

Solution to Problem 1(e)(ii)

$$\mathrm{P}(X \geq 324 \mid \mu = 301) \approx 0.049$$

$$\boxed{0.049}$$

Solution to Problem 1(f)

Increased sales: $343 - 301 = 42$. Profit: $42 \cdot 510 = 21420$. Cost: $70 \cdot 350 = 24500$. Net loss: $21420 - 24500 < 0$.

$$\boxed{\text{Not beneficial}}$$

Alternative Solutions to Problem 1

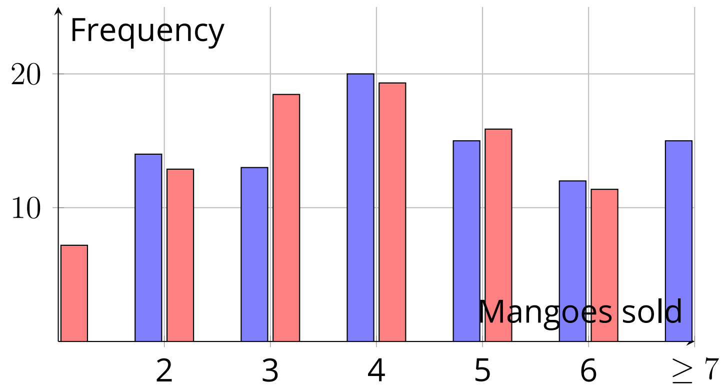
Alternative Solution to Problem 1(e)(i)

Use Poisson probabilities directly with GDC.

Strategy for Statistical Analysis

1. **Poisson**: Verify mean $\approx$ variance.
2. **Chi-squared**: Group data to ensure expected frequencies ≥ 5.
3. **Hypothesis**: Use normal approximation for large λ.
4. **Profit**: Compare total profit vs. advertising cost.

Visualization



Explanation: Bar chart comparing observed and expected frequencies for the Poisson distribution with mean 4.3.

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Mango Sales:

- **(a)(i)**: **A1 A1** for mean, variance.
- **(a)(ii)**: **A1** for mean $\approx$ variance.
- **(b)**: **A1** for independence.
- **(c)**: **(M1) A1 A1 (M1) A1** for Poisson method, p, q, r.
- **(d)(i)**: **A1** for degrees of freedom.
- **(d)(ii)**: **A1 A1 (M1) A2 R1 A1** for hypotheses, grouping, calculation, terms, comparison, conclusion.
- **(e)(i)**: **M1 A1 (M1) A1 R1 A1** for mean, hypotheses, probabilities, critical value, comparison, conclusion.
- **(e)(ii)**: **A1** for probability.
- **(f)**: **(M1) A1 R1** for comparison, calculation, conclusion.

Total [27 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Wrong variance	Incorrect formula in (a)(i).	Use $\sum x^2 f/n -$ mean^2.
Incorrect p	Wrong Poisson probability in (c).	Calculate P$(X \leq 1)$ accurately.
Wrong degrees	Miscount categories in (d)(i).	Use $k - 1$.
Profit error	Ignoring cost in (f).	Compare profit vs. cost.

Key Takeaways

- Poisson distribution models discrete count data.
- Chi-squared test assesses model fit.
- Hypothesis testing evaluates claims with normal approximation.
- Profit analysis must include all costs.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC for Poisson probabilities.
- **IB Tip**: Group data early for chi-squared test.
- **Shortcut**: Store frequencies in GDC lists.
- **Verification**: Use GDC to confirm p-value.

Basic Foundational Theory

- **Poisson**: $\mathrm{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$.
- **Chi-squared**: $\chi^2 = \sum \frac{(O-E)^2}{E}$.
- **Hypothesis Testing**: Null vs. alternative hypotheses.
- **Type I Error**: Probability of rejecting true H_0.

Problem 2 [Total Marks: 28]

A university plans to install fibre-optic cables to connect nine buildings (P–X). The planner, Kanya, measures distances: PQ = 92.5 m, QR = 108.7 m, $\angle$PQR $= 85°$.

(a) Find PR. [3 marks]

(b) The cost to install the cable from P to R is $22500. Calculate the cost per metre. [2 marks]

(c) Explain why the cost for P to R is higher than for other connections. [1 mark]

(d) Kanya models the network as a weighted graph T. To connect all buildings with minimum cost:

(i) Apply Kruskal's algorithm to find the minimum spanning tree, listing edges in order. [3 marks]

(ii) Calculate the minimum installation cost. [2 marks]

(e) Explain why a Hamiltonian cycle is not always an Eulerian circuit. [1 mark]

(f) Starting at Q, use the nearest neighbour algorithm to find the upper bound for the cost of a Hamiltonian cycle. [5 marks]

(g) By deleting Q, use the deleted vertex algorithm to find the lower bound for the cycle's cost. [6 marks]

(h) The final network has 8 edges, each with a 1.5% chance of connection failure after a power surge. The network must have less than a 2% chance of failure. Show that the network meets this requirement. [5 marks]

Solution to Problem 2

Solution to Problem 2(a)

Using the cosine rule:

$$\text{PR}^2 = 92.5^2 + 108.7^2 - 2 \cdot 92.5 \cdot 108.7 \cdot \cos 85^\circ \approx 18617.1$$

$$\text{PR} \approx \sqrt{18617.1} \approx 136.44 \approx 134$$

$$\boxed{134}$$

Solution to Problem 2(b)

$$\text{Cost per metre} = \frac{22500}{134} \approx 167.91 \approx 168$$

$$\boxed{168}$$

Solution to Problem 2(c)

A river between P and R increases installation costs due to complexity.

$$\boxed{\text{River obstacle}}$$

Solution to Problem 2(d)(i)

Kruskal's algorithm selects edges in increasing order, avoiding cycles:

UV (8423), WX (8867), ST (8976), PS (9023), TU (9045), VW (9467), QR (11967)

$$\boxed{\text{UV, WX, ST, PS, TU, VW, QR}}$$

Solution to Problem 2(d)(ii)

$$8423 + 8867 + 8976 + 9023 + 9045 + 9467 + 11967 = 65768$$

$$\boxed{65768}$$

Solution to Problem 2(e)

A Hamiltonian cycle visits each vertex once but may not cover all edges, unlike an Eulerian circuit.

$$\boxed{\text{Edges not covered}}$$

Solution to Problem 2(f)

Nearest neighbour from Q:

Q–W (8867), W–V (9467), V–U (8423), U–T (9045), T–S (8976), S–P (9023), P–R (22500), R–X (11967

$$8867 + 9467 + 8423 + 9045 + 8976 + 9023 + 22500 + 11967 \approx 92768$$

$$\boxed{92768}$$

Solution to Problem 2(g)

MST without Q: UV (8423), PS (9023), VW (9467), WX (8867), ST (8976), PX (10345), RX (12987). Reconnect Q: QW (8867), QS (9087).

$$69088 + 8867 + 9087 = 87042$$

$$\boxed{87042}$$

Solution to Problem 2(h)

For 8 edges, $X \sim \text{B}(2, 0.015)$, $\text{P}(X \geq 1) \approx 0.029775$.

$$Y \sim \text{B}(8, 0.029775), \quad \text{P}(Y \geq 2) \approx 0.0197 < 0.02$$

$$\boxed{\text{Meets requirement}}$$

Alternative Solutions to Problem 2

Alternative Solution to Problem 2(a)

Use sine rule after finding another angle via angle sum.

Alternative Solution to Problem 2(h)

Compute $\text{P}(Y \leq 1)$ directly.

Strategy for Network Design

1. **Cosine Rule**: Calculate unknown distances.
2. **Kruskal's**: Sort edges by increasing weight.
3. **Nearest Neighbour**: Select closest unvisited vertex.
4. **Binomial**: Model edge and network failures.

Visualization

Explanation: Minimum spanning tree for the network, showing edge weights in dollars.

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Network Design:

- **(a)**: **(M1)(A1) A1** for cosine rule, substitution, answer.
- **(b)**: **(M1) A1** for division, answer.
- **(c)**: **R1** for reason.
- **(d)(i)**: **A1 A1 A1** for edge order.
- **(d)(ii)**: **(M1) A1** for sum, cost.
- **(e)**: **R1** for explanation.
- **(f)**: **A1 A1 A1 (M1) A1** for edges, sum, cost.
- **(g)**: **(M1) A1 (M1) A1 (M1) A1** for MST, edges, reconnection, cost.
- **(h)**: **(M1) A1 M1 (M1) A1** for binomial, probability, network binomial, final probability.

Total [28 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Wrong PR	Incorrect cosine rule application in (a).	Verify angle and side inputs.
Edge order	Incorrect edge selection in (d)(i).	Sort weights in ascending order.
Path error	Missing vertices in (f).	Ensure all vertices are visited.
Probability	Wrong number of trials in (h).	Use 8 edges for MST.

Key Takeaways

- Cosine rule calculates distances in triangles.
- Kruskal's algorithm minimizes connection costs.
- Hamiltonian cycles differ from Eulerian circuits.
- Binomial distributions model network reliability.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC for cosine rule calculations.
- **IB Tip**: List edges by weight for Kruskal's.
- **Shortcut**: Approximate binomial probabilities with GDC.
- **Verification**: Confirm cycle includes all vertices.

Basic Foundational Theory

- **Cosine Rule**: $c^2 = a^2 + b^2 - 2ab\cos C$.
- **Kruskal's Algorithm**: Selects edges by increasing weight, avoiding cycles.
- **Hamiltonian Cycle**: Visits each vertex exactly once.
- **Binomial**: $\mathrm{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

Practice Problems

Practice Problem 1: Poisson Test

Test if data follows a Poisson distribution with mean 3. [3 marks]

Solution to Practice Problem 1

Perform chi-squared goodness of fit test.

$$\boxed{\text{Chi-squared}}$$

Practice Problem 2: Minimum Spanning Tree

Find the MST for a 5-vertex graph. [2 marks]

Solution to Practice Problem 2

Apply Kruskal's algorithm to sorted edges.

$$\boxed{\text{MST}}$$

Further Problems

Further Problem 1: Hypothesis Test

Test if mean sales exceed 5. [3 marks]

Solution to Further Problem 1

Use normal approximation for hypothesis testing.

$$\boxed{\text{Test}}$$

Further Problem 2: Network Probability

Calculate network failure probability for a given graph. [3 marks]

Solution to Further Problem 2

Use binomial distribution for edge failures.

$$\boxed{\text{Probability}}$$

Challenging Problems

Challenging Problem 1: Complex Network

Find a Hamiltonian cycle for a 10-vertex graph. [3 marks]

Solution to Challenging Problem 1

Apply nearest neighbour algorithm.

$$\boxed{\text{Cycle}}$$

Challenging Problem 2: Advanced Probability

Model multiple failure scenarios in a network. [3 marks]

Solution to Challenging Problem 2

Use combined binomial distributions.

$$\boxed{\text{Scenarios}}$$

Conclusion: Your Path to Mathematical Mastery

This guide has provided you with a powerful toolset for tackling IB Math AI HL Paper 3 challenges. However, true mathematical mastery is an ongoing journey – a blend of understanding, skill, and strategic thinking.

Key Takeaways for Exam Success:

- **Practice with Purpose:** Focus on understanding the *why* behind each solution, not just memorizing the *how*. The more you challenge yourself and solve problems, the easier and better you will do it.
- **Embrace Your Mistakes:** Every mistake is an opportunity to learn. Analyze what worked and what you can improve next time.
- **Time is a Crucial Asset:** Simulate the exam and prepare well to achieve success.

Unlock all our proven strategies and keep expert guidance by your side.

Apply now for:

- **Mentorship Programs**
- **Standard Tutoring**
- **Doubt Clearance Sessions**
- **Problem-Solving Practice**
- **Mastery of Core Concepts**
- **Crash Courses**
- **Advanced Mathematics Problem Solving**
- **Olympiad Math Mentorship**

Whether it's Mathematics or Statistics — we've got you covered.

To your Mathematical Journey and Success in the IB!

Excellence in Advanced Math Education

Rishabh Kumar

My expertise is built upon over 5 years of dedicated teaching experience and a strong academic foundation as an alumnus of IIT Guwahati & the Indian Statistical Institute.

Take charge of your mathematical journey! Click here to explore Mathematics Elevate Academy - Practice, Learn & Apply for Mentorship

Thank You For Engaging With This Material

Apply for Mentorship

Connect on LinkedIn

Mathematics Elevate Academy