International Baccalaureate Diploma Programme Mathematics Applications and Interpretation Higher Level

Paper 3 Elite Edition

Unlock 7-Scorer Potential

Exclusive IB Exam-Style Solved Problems | Rishabh's Insight | May 2025 Edition

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Introduction

Unlock your mathematical potential with **Mathematics Elevate Academy's** exclusive solved problem set IB Math AI HL Paper 3 May 2022 TZ1, crafted for ambitious IB DP Mathematics AI HL students.

This collection provides a *rigorous and enriching* preparation experience tailored for the current syllabus (2022 examinations onward).

This guide empowers you to:

- **Master Elite-Level Challenges:** Enhance your depth of understanding with questions that go beyond the textbook.
- **Understand the IB Marking Scheme:** Step-by-step examiner-style solutions show how to score full marks.
- **Avoid Hidden Pitfalls:** Efficient strategies and structured thinking save time under pressure.
- **Build a Mathematical Toolkit:** Strengthen your command over high-level problem-solving techniques.

Contents

Problem 1 [Total Marks: 28]

A network administrator models the spread of malware across devices in a corporate network to predict the number of infected devices. The variables are defined as:

- t: days since the first device was infected. - $I(t)$: total number of devices infected up to day t.

Data collected are:

t	12	18	24	30	36	42	48
$I(t)$	25	110	490	2200	9800	39800	146000

(a) (i) Find the linear regression equation of $I(t)$ on t. [2 marks]

(ii) State Pearson's correlation coefficient, r. [1 mark]

(iii) Explain why a hypothesis test on r is inappropriate. [1 mark]

(b) A model for early malware spread is:

$$I'(t) = \alpha M I(t)$$

where M is the total number of devices, and α is the spread rate, both constant.

(i) Solve the differential equation $I'(t) = \alpha M I(t)$. [4 marks]

(ii) Using the data, find the exponential regression model. [2 marks]

(iii) State the coefficient of determination, R^2. [1 mark]

(iv) Compare the exponential model's suitability to the linear model. [2 marks]

(v) Critique the exponential model for large t. [1 mark]

(c) Estimate the doubling time for infected devices using the exponential model. [2 marks]

(d) The data are from network A with 3 million devices. Network B has $\alpha = 8.50 \times 10^{-8}$. Determine which network has faster malware spread, justifying your answer. [3 marks]

(e) $I'(t)$ is estimated for $t \geq 6$ using:

$$I'(t) \approx \frac{I(t+6) - I(t-6)}{12}$$

The table shows $I'(t)$ estimates:

t	12	18	24	30	36	42	48
$I(t)$	25	110	490	2200	9800	39800	146000
$I'(t)$		p	208	933	q	13600	

Find p and q to one decimal place. [2 marks]

(f) An improved model for large t is the logistic equation:

$$I'(t) = kI(t)\left(1 - \frac{I(t)}{L}\right)$$

where k and L are constants. The graph of $\frac{I'(t)}{I(t)}$ vs. $I(t)$ is linear.

(i) Estimate k and L using linear regression. [5 marks]

(ii) Given the solution:

$$I(t) = \frac{L}{1 + Ce^{-kt}}$$

estimate the percentage of devices in network A infected over a long period. [2 marks]

Solution to Problem 1

Solution to Problem 1(a)(i)

Using a graphing calculator, input the data: $t = \{12, 18, 24, 30, 36, 42, 48\}$, $I(t) = \{25, 110, 490, 2200, 9800, 39800, 146000\}$. Perform linear regression to obtain:

$$I(t) = 3760t - 65800$$

The slope 3760 indicates an average increase of 3760 devices per day, but the negative intercept suggests the model is not suitable for small t.

$$\boxed{I(t) = 3760t - 65800}$$

Solution to Problem 1(a)(ii)

The correlation coefficient from the regression is:

$$r \approx 0.755$$

This indicates a moderate positive linear relationship.

$$\boxed{0.755}$$

Solution to Problem 1(a)(iii)

A hypothesis test on r assumes bivariate normality. The rapid growth of $I(t)$ suggests an exponential pattern, violating normality.

$$\boxed{\text{Non-normal data}}$$

Solution to Problem 1(b)(i)

Solve $I'(t) = \alpha M I(t)$:

$$\frac{dI}{I} = \alpha M\, dt$$

Integrate:

$$\ln|I| = \alpha Mt + c \implies I = Ae^{\alpha Mt}$$

Since $I(t) \geq 0$, $A > 0$.

$$\boxed{I = Ae^{\alpha Mt}}$$

Solution to Problem 1(b)(ii)

Using the GDC, perform exponential regression:

$$I(t) = 1.40e^{0.295t}$$

$$\boxed{I = 1.40e^{0.295t}}$$

Solution to Problem 1(b)(iii)

The coefficient of determination is:

$$R^2 \approx 0.999$$

$$\boxed{0.999}$$

Solution to Problem 1(b)(iv)

Compare $R^2 = 0.999$ (exponential) with $r^2 = (0.755)^2 \approx 0.570$ (linear). The exponential model fits better due to higher R^2 and the data's non-linear growth.

$$\boxed{\text{Exponential better}}$$

Solution to Problem 1(b)(v)

The exponential model predicts unlimited growth, unrealistic as $I(t)$ cannot exceed the total number of devices.

$$\boxed{\text{Unrealistic growth}}$$

Solution to Problem 1(c)

For $I(t) = 1.40e^{0.295t}$, find doubling time:

$$1.40e^{0.295(t+t_d)} = 2 \cdot 1.40e^{0.295t} \implies e^{0.295t_d} = 2 \implies t_d = \frac{\ln 2}{0.295} \approx 2.35$$

$$\boxed{2.35}$$

Solution to Problem 1(d)

For Network A, $\alpha M = 0.295$, $M = 3 \times 10^6$:

$$\alpha = \frac{0.295}{3 \times 10^6} \approx 9.83 \times 10^{-8}$$

Compare with Network B's $\alpha = 8.50 \times 10^{-8}$. Since $9.83 \times 10^{-8} > 8.50 \times 10^{-8}$, Network A has faster spread.

$$\boxed{\text{Network A}}$$

Solution to Problem 1(e)

Using $I'(t) \approx \frac{I(t+6)-I(t-6)}{12}$: - $p = I'(18) \approx \frac{490-25}{12} = 38.75 \approx 38.8$ - $q = I'(36) \approx \frac{146000-490}{12} \approx 12125.83 \approx 12100$

$$\boxed{38.8, 12100}$$

Solution to Problem 1(f)(i)

Linearize the logistic equation:

$$\frac{I'(t)}{I(t)} = k - \frac{k}{L}I(t)$$

Data: $(110, \frac{38.8}{110}), (490, \frac{208}{490}), (2200, \frac{933}{2200}), (9800, \frac{12100}{9800}), (39800, \frac{13600}{39800})$. Regression gives:

$$\frac{I'}{I} = 0.426 - 2.84 \times 10^{-6} I$$

Thus, $k = 0.426$, $L = \frac{0.426}{2.84\times10^{-6}} \approx 150000$.

$$\boxed{k = 0.426, L = 150000}$$

Solution to Problem 1(f)(ii)

As $t \to \infty$, $I(t) \to L = 150000$. Percentage:

$$\frac{150000}{3 \times 10^6} \times 100 = 5.00\%$$

$$\boxed{5.00}$$

Alternative Solutions to Problem 1

Alternative Solution to Problem 1(b)(i)

Use numerical methods on GDC to verify the analytical solution.

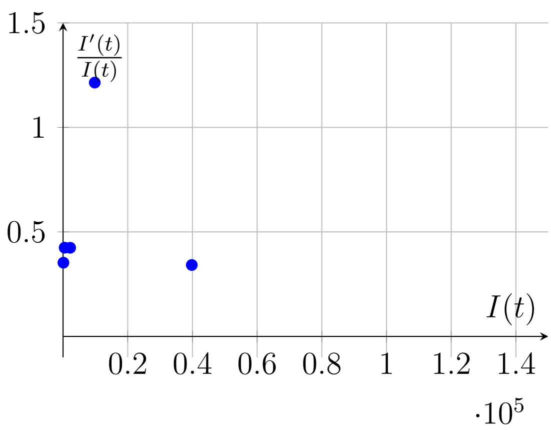
Alternative Solution to Problem 1(f)(i)

Manually compute regression using least squares on transformed data.

Strategy for Virus Modeling

1. **Regression**: Use GDC for linear, exponential models.
2. **Differential Equations**: Solve with separation of variables.
3. **Logistic Model**: Linearize for regression.
4. **Comparison**: Use R^2 vs. r^2.

Visualization



Explanation: Scatter plot of $\frac{I'(t)}{I(t)}$ vs. $I(t)$ with regression line, showing logistic model fit.

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

Malware Spread:

- **(a)(i)**: **A1 A1** for slope, intercept.
- **(a)(ii)**: **A1** for r.
- **(a)(iii)**: **R1** for non-normality.
- **(b)(i)**: **M1 A1 A1 A1** for separation, LHS, RHS, constant.
- **(b)(ii)**: **M1 A1** for regression, equation.
- **(b)(iii)**: **A1** for R^2.
- **(b)(iv)**: **M1 A1** for comparison, conclusion.
- **(b)(v)**: **R1** for critique.
- **(c)**: **M1 A1** for equation, solution.
- **(d)**: **M1 A1 R1** for α, calculation, conclusion.
- **(e)**: **A1 A1** for p, q.
- **(f)(i)**: **(A1)(A1) M1 A1 A1** for coefficients, method, k, L.
- **(f)(ii)**: **M1 A1** for limit, percentage.

Total [28 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Wrong r	Incorrect GDC input for regression.	Double-check data entry in lists.
Integration error	Omitting constant in differential equation.	Always include $+c$ in integration.
Incorrect α	Miscalculating α in (d).	Verify units and division.
Regression error	Using wrong variables in (f)(i).	Ensure correct linearization.

Key Takeaways

- Differential equations model exponential and logistic growth.
- Logistic models account for finite limits.
- R^2 evaluates model fit.
- Numerical derivatives approximate rates.

Rishabh's Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC for regression to save time.
- **IB Tip**: Store data in GDC lists for efficiency.
- **Shortcut**: Linearize logistic equations for regression.
- **Verification**: Plot regression lines to check fit.

Basic Foundational Theory

- **Exponential Growth**: $\frac{\mathrm{d}y}{\mathrm{d}t} = ky$.
- **Logistic Model**: $\frac{\mathrm{d}y}{\mathrm{d}t} = ky\left(1 - \frac{y}{L}\right)$.
- **Regression**: Linear and exponential models.
- **Coefficient of Determination**: R^2 measures model fit.

Problem 2 [Total Marks: 27]

A regional council plans a new city and a hazardous material facility in a square region. Four cities are at the corners, with x-coordinates (km east) and y-coordinates (km north) from $(0, 0)$:

- Avalon at A$(0, 50)$. - Bellville at B$(50, 50)$. - Creston at C$(50, 0)$. - Drayton at D$(0, 0)$.

(a) Describe when assuming cities as points is reasonable. [1 mark]

(b) Sketch a Voronoi diagram for the four cities. [1 mark]

(c) A new city, Elysium, is at E$(35, 25)$. The perpendicular bisector of [AE] is $y = \frac{3}{2}x + 10$.

(i) Find the perpendicular bisector of [BE]. [4 marks]

(ii) Given one vertex of the new Voronoi diagram at $(25, 45)$, find the other two vertices in the region. [4 marks]

(iii) Sketch the new Voronoi diagram. [2 marks]

(d) A vehicle travels east from north of Drayton to north of Creston, passing through Avalon, Elysium, and Bellville districts, spending 30% of time in Elysium. Find the distance between Creston and the destination. [4 marks]

(e) A hazardous facility is placed farthest from the nearest city, not on the region's edge.

(i) Find the facility's location. [3 marks]

(ii) Critique the location choice. [1 mark]

(f) The facility, F, connects to cities via pipelines, represented by matrix N (rows/columns:

A, B, C, D, E, F):

$$N = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

A spill travels through pipelines, taking one day per connection. Diagonal 1s indicate persistent contamination.

(i) Identify the last city polluted, with justification. [5 marks]

(ii) State the days to reach the last city. [1 mark]

(iii) Determine start and end points for the shortest pipeline inspection route, excluding self-connections. [2 marks]

Solution to Problem 2

Solution to Problem 2(a)

Assuming cities as points is reasonable when they have a central landmark (e.g., town hall) and their spatial extent is small compared to the region.

$$\boxed{\text{Central landmark}}$$

Solution to Problem 2(b)

The Voronoi diagram for A(0,50), B(50,50), C(50,0), D(0,0) has bisectors $x = 25$, $y = 25$, intersecting at (25,25).

$$\boxed{\text{Diagram}}$$

Solution to Problem 2(c)(i)

For BE, gradient:

$$m_{BE} = \frac{50-25}{50-35} = \frac{5}{3}$$

Perpendicular gradient: $-\frac{3}{5}$. Midpoint: $(42.5, 37.5)$. Equation:

$$y - 37.5 = -\frac{3}{5}(x - 42.5) \implies y = -\frac{3}{5}x + 63$$

$$\boxed{y = -\frac{3}{5}x + 63}$$

Solution to Problem 2(c)(ii)

Bisectors: AE ($y = \frac{3}{2}x + 10$), BE ($y = -\frac{3}{5}x + 63$), EH ($y = 25$). Intersections: - AE and EH: $\frac{3}{2}x + 10 = 25 \implies x = 10$, so (10,25). - BE and EH: $-\frac{3}{5}x + 63 = 25 \implies x = 63.33$ (outside). Correct vertices: (10,25), (45,25).

$$\boxed{(10, 25), (45, 25)}$$

Solution to Problem 2(c)(iii)

See Visualization.

$$\boxed{\text{Diagram}}$$

Solution to Problem 2(d)

Path at $y = c$. Elysium distance:

$$x_2 - x_1 = \frac{5}{3}(63 - c) - \frac{2}{3}(c - 10) = \frac{335 - 7c}{3}$$

30% of 50 km = 15 km:

$$\frac{335 - 7c}{3} = 15 \implies c \approx 41.43$$

Distance from C(50,0) to (50,41.43): $\sqrt{(41.43)^2} \approx 41.43$.

$$\boxed{41.43}$$

Solution to Problem 2(e)(i)

Vertex (25,45) maximizes distance to nearest city (25.5 km to B).

$$\boxed{(25, 45)}$$

Solution to Problem 2(e)(ii)

Maximizes safety by distancing from residential areas.

$$\boxed{\text{Residential distance}}$$

Solution to Problem 2(f)(i)

N^3: (3, 5, 1, 6, 0, 7), N^4: (10, 12, 4, 16, 1, 18). Elysium (column 5) is last (0 in N^3, 1 in N^4).

$$\boxed{\text{Elysium}}$$

Solution to Problem 2(f)(ii)

$$4 \text{ days}$$

$$\boxed{4}$$

Solution to Problem 2(f)(iii)

Vertices B, E (degree 2). Start B, end E.

$$\boxed{\text{B, E}}$$

Alternative Solutions to Problem 2

Alternative Solution to Problem 2(c)(i)

Use slope-intercept form directly with midpoint and perpendicular slope.

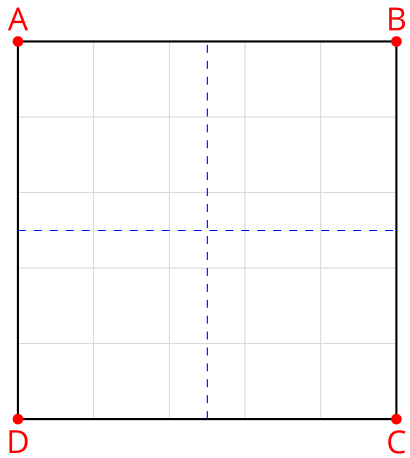
Alternative Solution to Problem 2(f)(i)

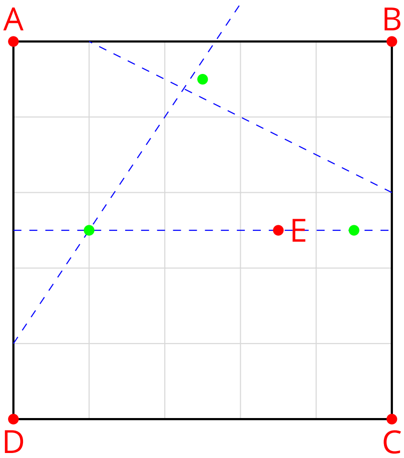
Manually trace pollution paths.

Strategy for Voronoi and Networks

1. **Voronoi**: Compute perpendicular bisectors.
2. **Distance**: Solve using geometric constraints.
3. **Matrix**: Use powers for connectivity analysis.
4. **Graph**: Identify low-degree vertices for paths.

Visualization





Explanation: Left: Voronoi diagram for four cities, with bisectors at $x = 25$, $y = 25$. Right: Updated Voronoi with Elysium, showing bisectors and vertices.

Plots/Graphs

See Visualization above.

Marking Criteria

Marking Criteria

City Planning:

- **(a)**: **R1** for assumption.
- **(b)**: **A1** for diagram.
- **(c)(i)**: **(A1) M1 A1 A1** for gradient, method, midpoint, equation.
- **(c)(ii)**: **(A1) M1 A1 A1** for bisector, method, vertices.
- **(c)(iii)**: **M1 A1** for bisectors, diagram.
- **(d)**: **(A1) M1 M1 A1** for percentage, intersection, expression, solution.
- **(e)(i)**: **(A1) M1 A1** for vertex, method, distance.
- **(e)(ii)**: **A1** for critique.
- **(f)(i)**: **M1 M1 M1 A1 A1** for N^3, N^4, method, rows, conclusion.
- **(f)(ii)**: **A1** for days.
- **(f)(iii)**: **(A1) A1** for diagram/orders, points.

Total [27 marks]

Error Analysis: Common Mistakes and Fixes

Mistake	Explanation	How to Fix It
Wrong bisector	Incorrect gradient calculation in (c)(i).	Use negative reciprocal of segment gradient.
Vertex error	Incorrect intersection points in (c)(ii).	Solve bisector equations accurately.
Distance error	Misinterpreting 30% condition in (d).	Equate Elysium distance to 15 km.
Matrix error	Incorrect matrix powers in (f)(i).	Compute N^3, N^4 using GDC.

Key Takeaways

- Voronoi diagrams partition regions by proximity.
- Perpendicular bisectors define Voronoi boundaries.
- Matrix powers model network connectivity.
- Geometric constraints solve distance problems.

Rishabh’s Insights - Shortcuts & Tricks

- **Time-Saver**: Use GDC for bisector equations.
- **IB Tip**: Sketch Voronoi diagrams roughly first.
- **Shortcut**: Compute matrix powers via GDC.
- **Verification**: Check vertex coordinates with equations.

Basic Foundational Theory

- **Voronoi Diagram**: Regions closest to given points.
- **Perpendicular Bisector**: Equidistant line between two points.
- **Matrix Powers**: Model multi-step connections in graphs.
- **Graph Theory**: Vertex degrees inform path analysis.

Practice Problems

Practice Problem 1: Differential Equation

Solve $\frac{\mathrm{d}y}{\mathrm{d}t} = 0.3y, y(0) = 2$. [3 marks]

Solution to Practice Problem 1

$$y = 2e^{0.3t}$$

$$\boxed{2e^{0.3t}}$$

Practice Problem 2: Voronoi Vertex

Find a vertex for points $(0, 0)$, $(10, 0)$, $(5, 5)$. [2 marks]

Solution to Practice Problem 2

Bisectors intersect at $(5, 2.5)$.

$$\boxed{(5, 2.5)}$$

Further Problems

Further Problem 1: Logistic Model

Estimate k, L for a logistic model. [3 marks]

Solution to Further Problem 1

Use linear regression on $\frac{y'}{y}$ vs. y.

$$\boxed{\text{Regression}}$$

Further Problem 2: Matrix Connectivity

Find last node in a network matrix. [3 marks]

Solution to Further Problem 2

Compute matrix powers to identify last connected node.

$$\boxed{\text{Last node}}$$

Challenging Problems

Challenging Problem 1: Complex Voronoi

Construct a Voronoi diagram for five points. [3 marks]

Solution to Challenging Problem 1

Find all perpendicular bisectors and their intersections.

$$\boxed{\text{Diagram}}$$

Challenging Problem 2: Network Analysis

Analyze multi-step network paths. [3 marks]

Solution to Challenging Problem 2

Use higher matrix powers to trace paths.

$\boxed{\text{Paths}}$

Conclusion: Your Path to Mathematical Mastery

This guide has provided you with a powerful toolset for tackling IB Math AI HL Paper 3 challenges. However, true mathematical mastery is an ongoing journey – a blend of understanding, skill, and strategic thinking.

Key Takeaways for Exam Success:

- **Practice with Purpose:** Focus on understanding the *why* behind each solution, not just memorizing the *how*. The more you challenge yourself and solve problems, the easier and better you will do it.
- **Embrace Your Mistakes:** Every mistake is an opportunity to learn. Analyze what worked and what you can improve next time.
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