International Baccalaureate Diploma Programme Mathematics Analysis and Approaches Higher Level

Master Volume of Revolution

Unlock 7-Scorer Potential

Concept | Practice Set | April 2025 Edition

Mathematics Elevate Academy

Excellence in Advanced Math Education

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Disclaimer

Preface

This guide is designed for ambitious learners aiming for excellence—those who wish to truly ***understand****, not merely memorize.* Crafted with precision and clarity, the content within reflects the standards of elite mathematical training. Whether you're preparing for your IB examinations or seeking to deepen your foundation in calculus, this resource offers a step-by-step exploration, richly illustrated with:

- Carefully structured **conceptual breakdowns**
- **Solved examples** with insightful commentary
- Handpicked **IB exam style problems**
- A thoughtfully curated set of **practice and challenge problems**

As a global educator specializing in advanced mathematics and statistical thinking, I am tailored to mentor students across continents to achieve the coveted **Level 7** in IB Math AA HL. This note is a distilled reflection of that experience—crafted not just to teach, but to *inspire*.

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Introduction

Unlock your mathematical potential with **Mathematics Elevate Academy's** master selected topics series, crafted for ambitious IB DP Mathematics AA HL students.

The **Volume of Revolution** is a cornerstone of integral calculus, enabling the calculation of volumes of complex solids formed by rotating curves around axes. This technique is not just a computational tool but a gateway to understanding symmetry, approximation, and the power of calculus in modeling real-world phenomena.

In the **IB Mathematics: Analysis and Approaches HL** curriculum, mastering volume of revolution requires a deep grasp of integration techniques, visualization of 3D solids, and strategic problem-solving. This guide equips you with the tools to tackle IB exam questions with confidence.

Volume of Revolution

A Comprehensive Guide for IB Math AA HL

Core Concepts

Theoretical Framework

The volume of a solid formed by rotating a region around an axis is computed using:

- **Disc/Washer Method**:

$$V = \pi \int_a^b [f(x)^2 - g(x)^2] \, dx \quad \text{(for rotation about the x-axis)}$$

$$V = \pi \int_c^d [f(y)^2 - g(y)^2] \, dy \quad \text{(for rotation about the y-axis)}$$

- **Shell Method**:

$$V = 2\pi \int_a^b x \cdot h(x) \, dx \quad \text{(for rotation about the y-axis)}$$

$$V = 2\pi \int_c^d y \cdot h(y) \, dy \quad \text{(for rotation about the x-axis)}$$

Key Considerations

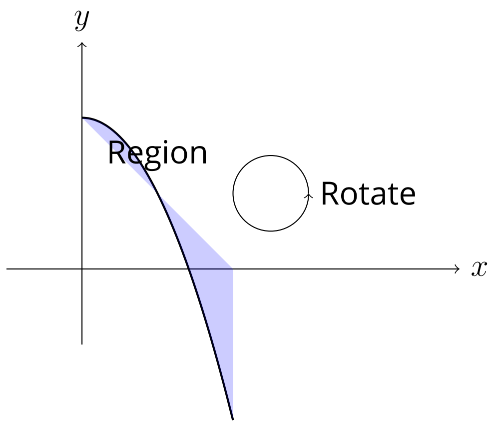
- **Axis of Rotation**: Choose between disc/shell based on the axis (e.g., shell method is often simpler for y-axis rotations).
- **Bounded Regions**: Ensure the region is clearly defined by curves and boundaries.

- **Adjustments for Non-Standard Axes**: For axes like $x = k$ or $y = k$, modify the radius formula (e.g., $r = |x - k|$).

Applications

- Engineering: Designing containers, turbine blades.
- Physics: Calculating volumes of celestial bodies or fluid dynamics.
- Economics: Modeling rotational symmetry in production costs.

Visualizing Volume of Revolution



How does the summation of infinitesimal discs/shells reflect the concept of *approximation* in calculus? Discuss the philosophical implications of using limits to model physical volumes.

Practice Example

Problem: Find the volume of the solid formed by rotating $y = x^2$ from $x = 0$ to $x = 1$ about the x-axis.

Solution: Use the disc method:

$$V = \pi \int_0^1 (x^2)^2 \, dx = \pi \int_0^1 x^4 \, dx = \frac{\pi}{5} \text{ units}^3$$

Solved Problems

Example 1: Disc Method (x-axis)

Problem: Rotate $y = \sqrt{x}$ from $x = 0$ to $x = 4$ about the x-axis.

Solution:

$$V = \pi \int_0^4 (\sqrt{x})^2 \, dx = \pi \int_0^4 x \, dx = 8\pi \text{ units}^3$$

Example 2: Shell Method (y-axis)

Problem: Rotate $y = x^2$ from $x = 0$ to $x = 2$ about the y-axis.

Solution:

$$V = 2\pi \int_0^2 x \cdot x^2 \, dx = 2\pi \int_0^2 x^3 \, dx = 8\pi \text{ units}^3$$

Example 3: Bounded Regions

Problem: Find the volume of the region bounded by $y = x$, $y = 2$, and $x = 0$, rotated about the x-axis.

Solution:

$$V = \pi \int_0^2 (2^2 - x^2) \, dx = \frac{16\pi}{3} \text{ units}^3$$

Verify integrals using Wolfram Alpha:

`Integrate[Pi*(`$2^2 - x^2), x, 0, 2]$

IB Exam-Style Problems

Problem 1 (Paper 1, 6 marks)

The region under $y = x^3$ from $x = 0$ to $x = 1$ is rotated about the x-axis.

(a) Set up the integral for the volume. *[2 marks]*

(b) Calculate the volume. *[3 marks]*

(c) Interpret the result as a physical volume. *[1 mark]*

Problem 2 (Paper 2, 8 marks)

The region bounded by $y = \sqrt{x}$, $x = 1$, $x = 4$, and $y = 0$ is rotated about the y-axis.

(a) Use the shell method to set up the integral. *[3 marks]*

(b) Find the volume. *[4 marks]*

(c) Verify using the disc method. *[1 mark]*

Problem 3 (Paper 1, 7 marks)

The region between $y = x^2$ and $y = 2x$ is rotated about the x-axis.

(a) Find the points of intersection. *[2 marks]*

(b) Set up the integral for the volume. *[2 marks]*

(c) Calculate the volume. *[3 marks]*

Challenge Problems

1. **Non-Standard Axis**: Find the volume when $y = e^x$, $x = 0$ to $x = 1$, is rotated about $y = 2$.
2. **Parametric Curves**: For $x(t) = t$, $y(t) = t^2$, $t = 0$ to $t = 1$, find the volume about the x-axis.
3. **Composite Regions**: Find the volume of the region between $y = x^2$ and $y = \sqrt{x}$, rotated about $x = -1$.
4. **Sphere Volume**: Derive the volume of a sphere using the disc method.

Summary Table

Method	Formula	Use Case
Disc (x-axis)	$\pi \int [f(x)]^2 \, dx$	Rotation about x-axis
Washer (x-axis)	$\pi \int [f(x)^2 - g(x)^2] \, dx$	Bounded regions
Shell (y-axis)	$2\pi \int x f(x) \, dx$	Rotation about y-axis

Quick Quiz

1. What is the disc method formula for rotation about the y-axis?
2. Set up the integral for $y = x$, $x = 0$ to $x = 1$, about the y-axis (shell method).
3. If $y = x^2$, $x = 0$ to $x = 1$, is rotated about $x = 1$, what is the radius?

Conclusion

Pro Tip: Always sketch the region and axis of rotation to choose the optimal method (disc vs. shell).

Final Thought: Volume of revolution is not just calculus—it's a bridge between algebraic precision and geometric intuition.

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