Graph & Transformation

$\Rightarrow$ $y = f(x)$ (Given/ Known)

then,

① $y = f(x+a)$ will be graph of backward shift by $(-a)$.

Similarly,

② $y = f(x-a)$ will be graph of forward shift by a.

③ $y = f(x) + a$ will be upward shift by a.

Similarly,

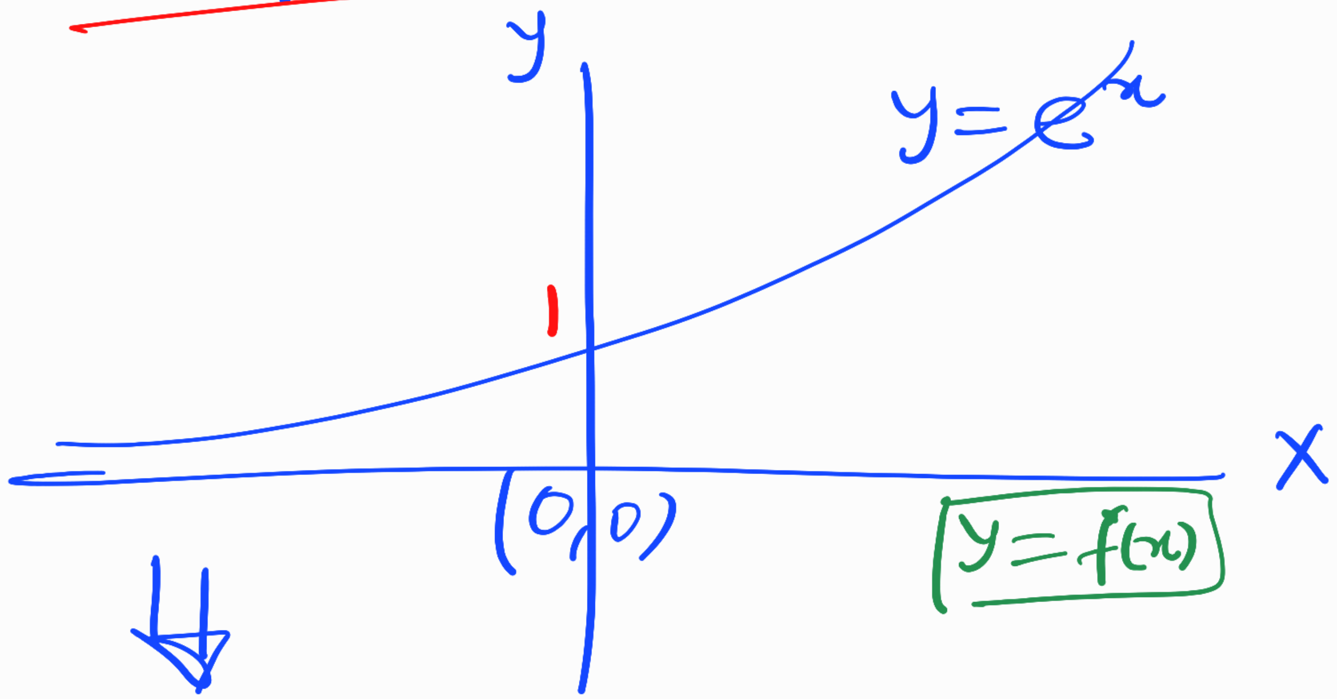
④ $y = f(x) - a$ will be downward shift by a.

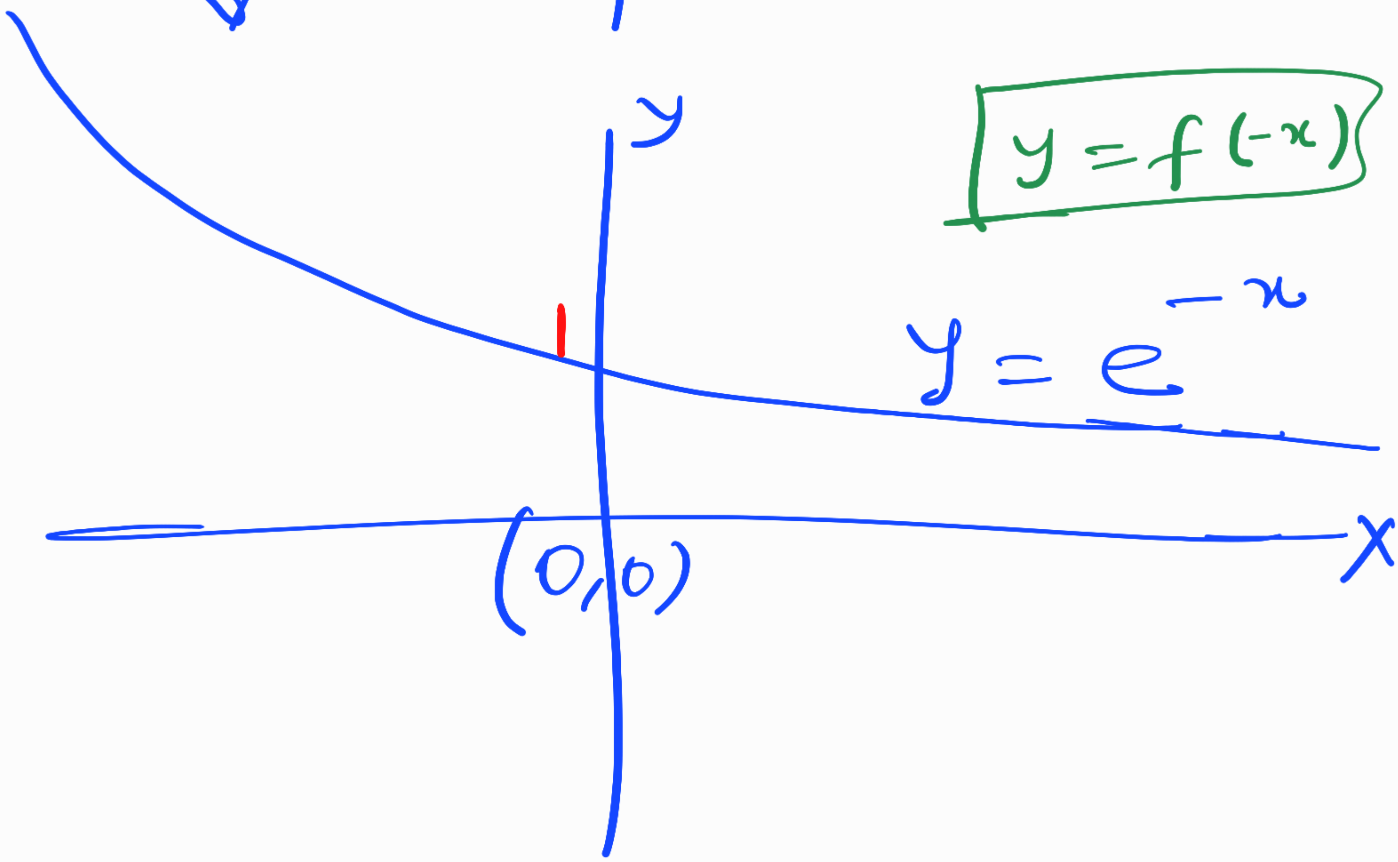
⑤ $y = K f(x)$, K - constant

Graph will be scaled by K factor.

Graph of some Standard functions :-

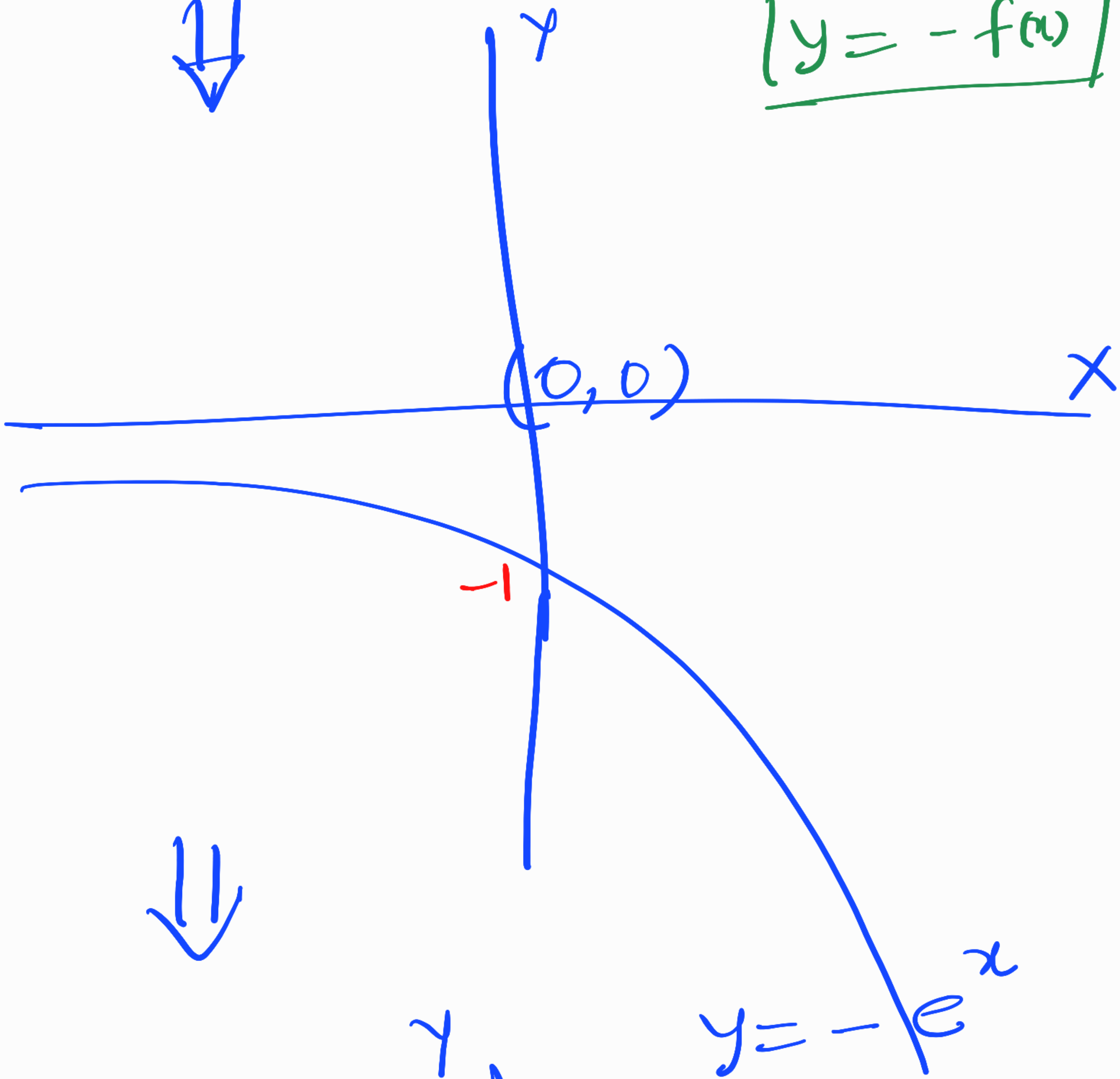
① Exponential



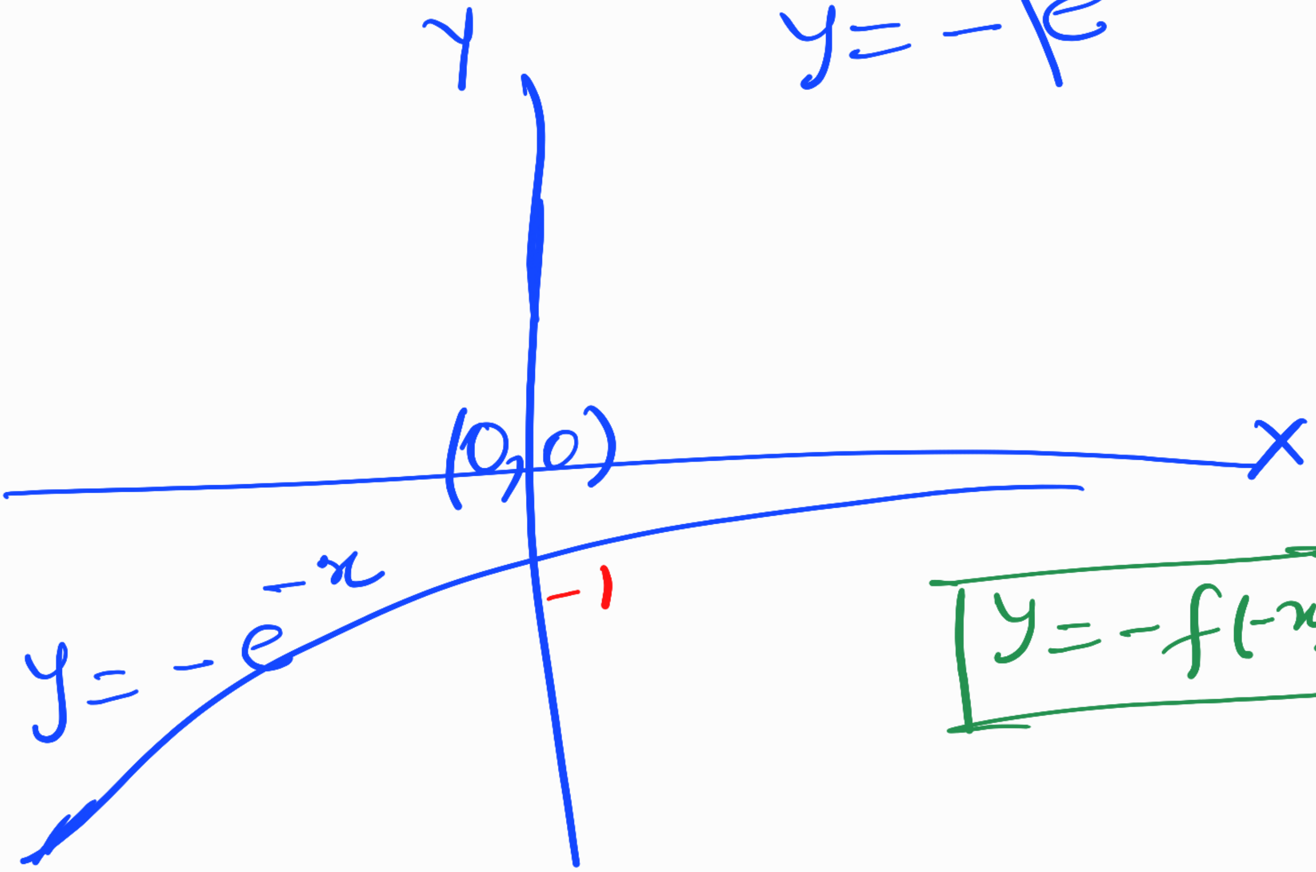


$\Downarrow$

$$\boxed{y = -f(x)}$$



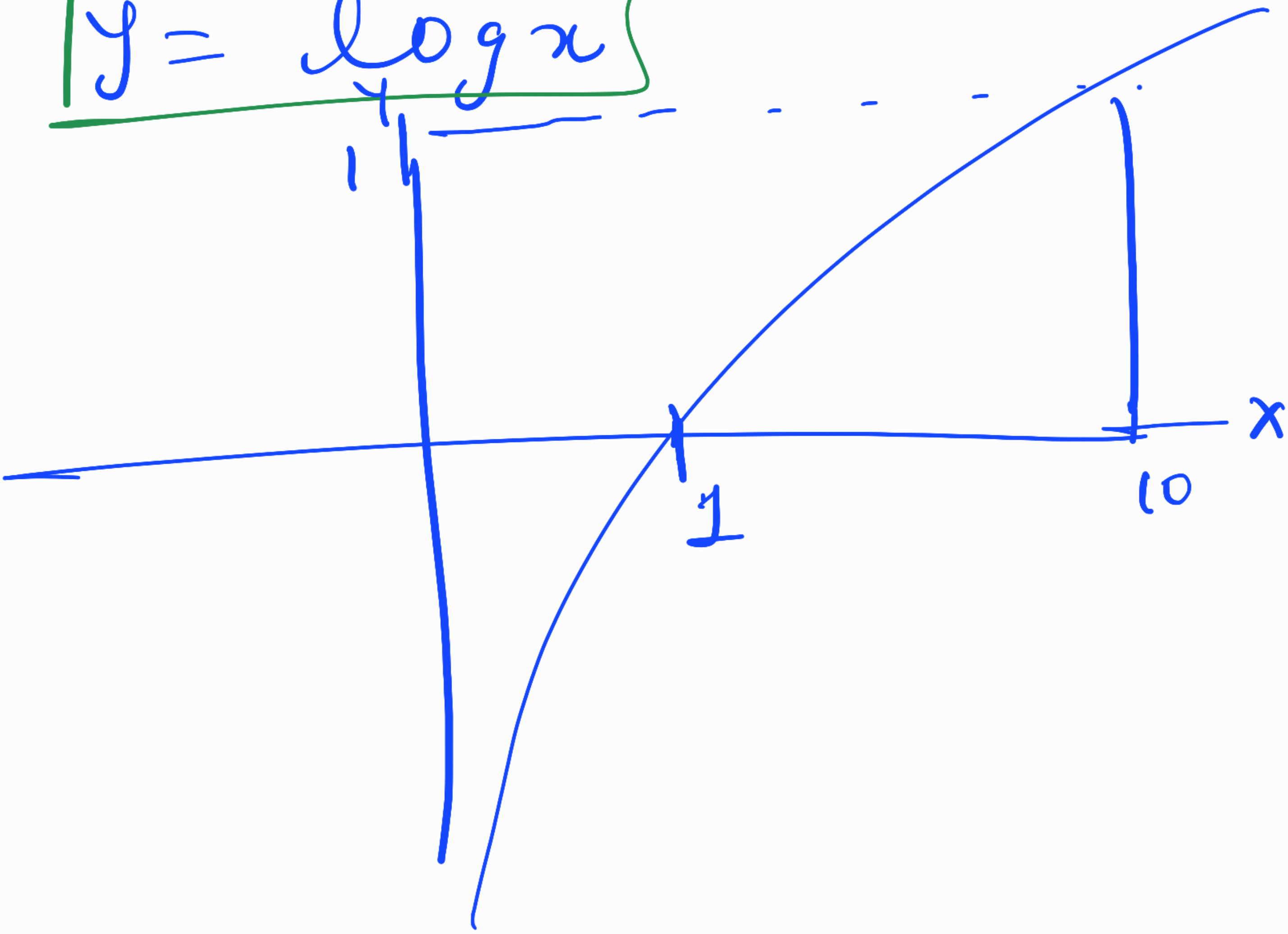
$\Downarrow$



$$\boxed{y = -f(-x)}$$

② Logarithm.

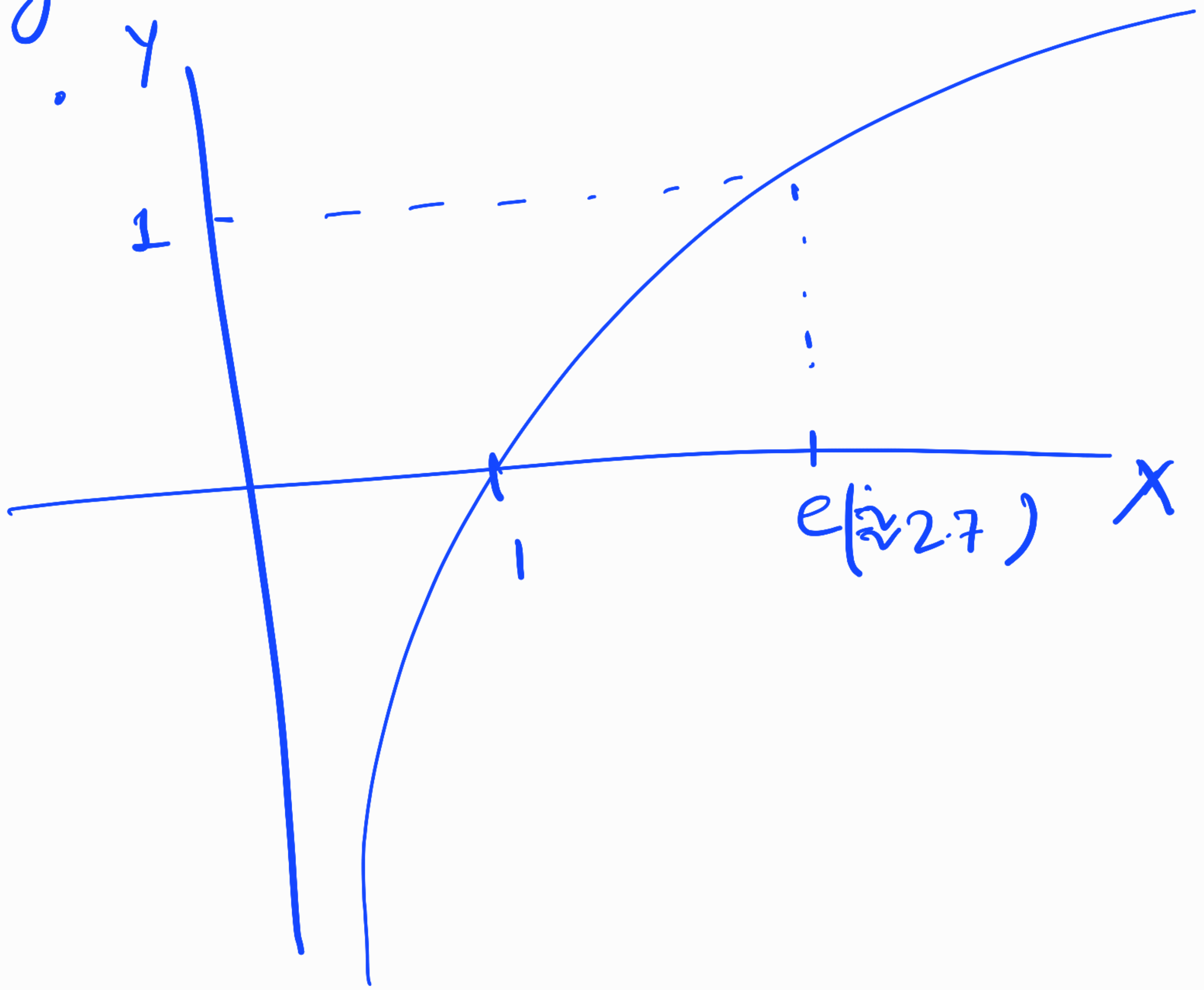
$\boxed{y = \log x}$

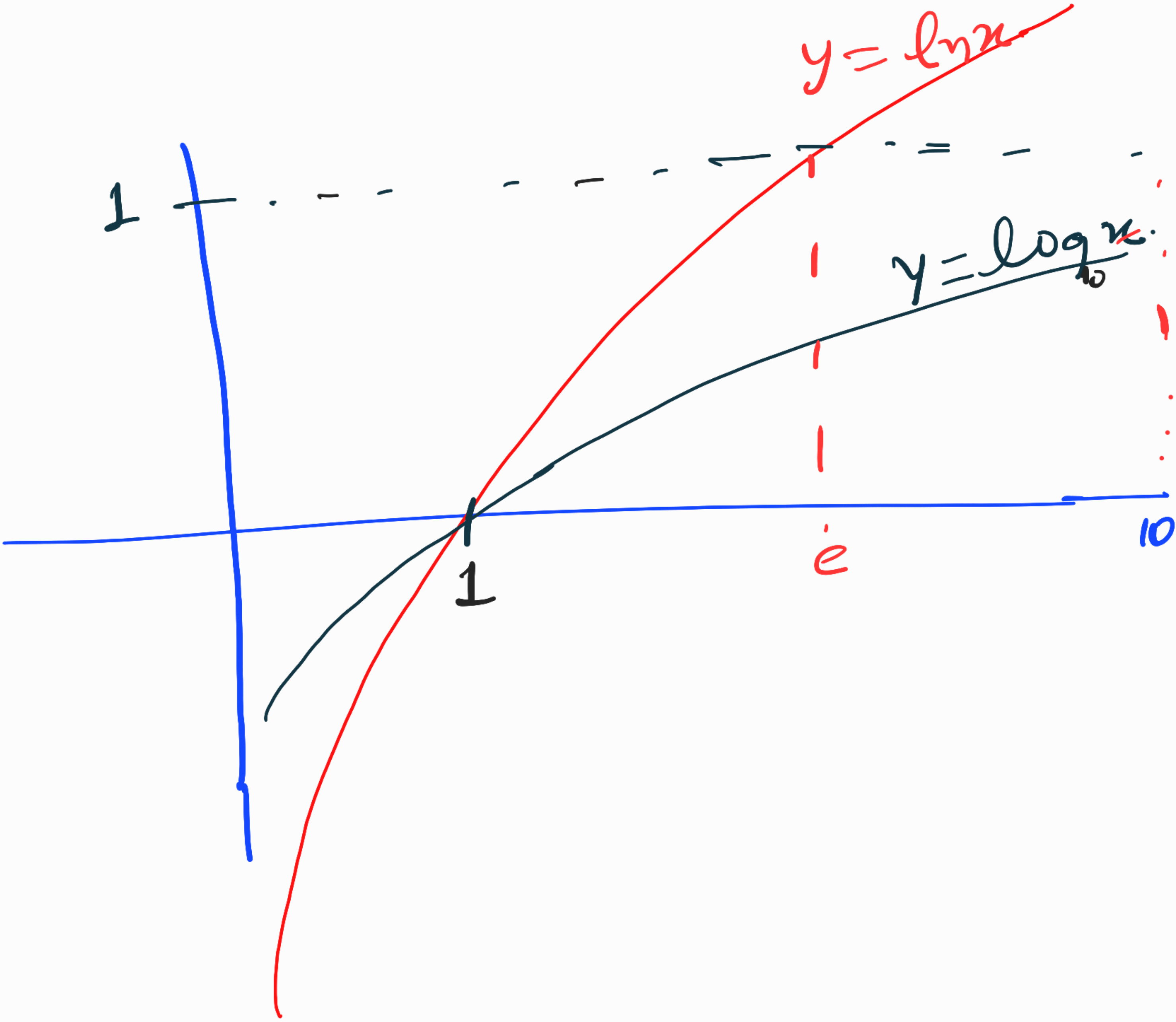


$y = \log x$ means base is 10 by default.

$y = \log_{10} x$.

$y = \ln x$ is natural logarithm where base is e.





Log : Definition

If $a^x = N$

$\Rightarrow \log_a N = x$

Properties of a log:-

For $\log_b a$:

1) $a > 0$
2) $b > 0$
3) $b \neq 1$

Compulsory (Necessary) otherwise log will not be defined.

$$\log_b a \begin{cases} = 0 & \text{if } a = 1 \\ = +ve & \text{if } \begin{cases} a > 1 \ \& \ b > 1 \\ \text{OR} \\ 0 < a < 1 \ \& \ 0 < b < 1 \end{cases} \end{cases}$$

$$\log_b a = -ve, \text{ if } \begin{cases} a > 1 \ \& \ 0 < b < 1 \\ \text{OR} \\ 0 < a < 1, \ b > 1 \end{cases}$$

$$\underline{\underline{a^x = N}} \Rightarrow \log_a N = x$$

$$a^x \cdot b^x = (ab)^x$$

Similarly,

$$\log a + \log b = \log ab$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$\log a \Rightarrow$ By default base is 10 for $\log a$.

Thus, $\log_{10} a = \log a$

Rule of change of base :-

$$\log_b a = \frac{\log_c a}{\log_c a}$$

for some c, $c \neq 1$

as log is not defined for base 'c' = 1.

$$\therefore \log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$$

$$\Rightarrow \log_b a^p = p \cdot \log_b a$$

Proof :-

We know,

$$a^x = N$$

$$\Rightarrow \log_a N = x$$

So, $\log_b a^p = y$ (Let)

$$b^y = a^p \quad \text{——} \quad (1)$$

Let, $\log_b a = y_1$

$\Rightarrow b^{y_1} = a \longleftrightarrow$ ②

Using ①,

we have

$$b^y = a^P$$

$$(b^{y_1})^P = (a)^P \quad (\text{from } ②)$$

$$b^{y_1 \cdot P} = a^P = b^y$$

So, $y = y_1 \cdot P$

Thus,

$$\log_b a^p = p \log_b a.$$

as we assumed,

$$y = \log_b a^p$$

$$y_1 = \log_b a$$

Note:

If $a^x = a^y$

$\Rightarrow x = y.$? ✓ ✗

$a^x = a^y \Rightarrow x = y$ (✓ ✗)

Answer : ✗

Correct except

when $\boxed{a = 1}$ or $\boxed{a = 0}$

As;

$(1^5 = 1^4) = 1$

$\Rightarrow 4 \neq 5$

$0^4 = 0^7 = 0$

Sample problem :-

Domain of $\sqrt{\log_x(\cos 2\pi x)}$?

Soln :- Conditions :-

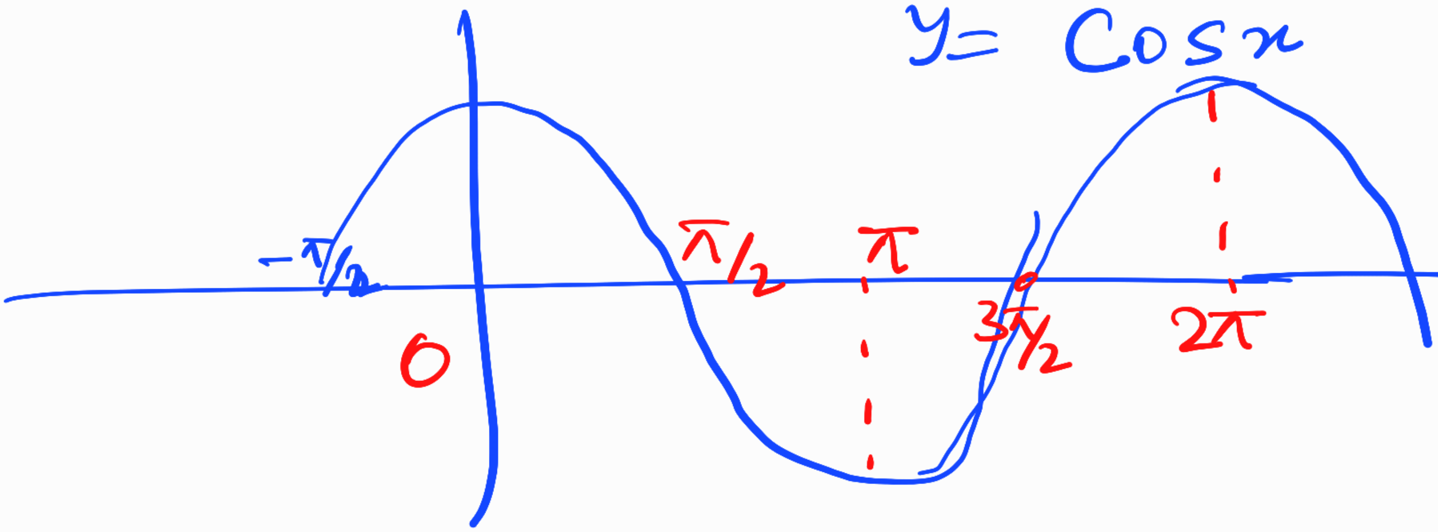
$$\log_x(\cos 2\pi x) \geq 0$$

$$x > 0$$

$$x \neq 1$$

$$\cos(2\pi x) > 0$$

$y = \cos x$



For $\cos(2\pi x) > 0$

So, $2\pi x \in$

$$\dots \cup \left(-\pi/2, \pi/2\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) \cup \dots$$

So,

$$x \in \dots \left(-1/4, 1/4\right) \cup \left(3/4, 5/4\right) \cup \dots$$

Other conditions

$x > 0$

$x \neq 1$

$\log_x(\cos 2\pi x) \geq 0$

$$\log_x(\cos 2\pi x) = 0$$

when $\cos 2\pi x = 1$

$\cos 2\pi x = \cos 2n\pi$

$n \in Z$

So, $x = n$

$$\log_x(\cos 2\pi x) > 0$$

when $x > 1$ & $\cos 2\pi x > 1$

or

$0 < x < 1$ & $0 < \cos 2\pi x < 1$

But, $\cos 2\pi x > 1$

So, only possibility $0 < x < 1$

Thus, All criteria's are :-

$$x > 0$$

$$0 < x < 1$$

$$x \neq 1$$

$$x = \ldots\left(-\tfrac{1}{4}, \tfrac{1}{4}\right) \cup \left(\tfrac{3}{4}, \tfrac{5}{4}\right) \ldots$$

$$x = n \ , \ n \in Z$$

Hence,

$$x = \left(0, \tfrac{1}{4}\right) \cup \left(\tfrac{3}{4}, 1\right)$$

$$\cup \ N \setminus \{1\}$$

$N \in$ Natural Number

Don't include 1.

Domain :- Where function can be defined.

Range :- Set of values which are output of the function

$$y = f(\underline{x})$$

$\uparrow$

Domain $\checkmark$

Range: $\checkmark$

Range can be computed by the domain of $f^{-1}(x)$

i.e. Solve for x to get $f^{-1}(x)$.

One - to - One / Injective function :

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2.$$

Onto / surjective function : Codomain = Range

Asymptotes of $y = \frac{f(x)}{g(x)}$

- Horizontal
  when $g(x) = 0$.
- vertical
  solve for x in terms of y then, denominator $= 0$.

$$\boxed{y = f(x)}$$

has maximum / minimum / saddle / Inflection point when

$f'(x) = 0.$

$f''(x) > 0$ minimum

$f''(x) < 0$ maximum

$f''(x) = 0 \rightarrow$ point of Inflection

$f'(x) > 0$: function is increasing & vice-versa

$$f(x) = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right]$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right]$$

$\Rightarrow \quad f(x) = 0$

$\Rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$ax^2 + bx + c = 0$

$2ax + b = 0$

$x = \dfrac{-b}{2a}$.

$y = \dfrac{4ac - b^2}{4a}$.

$\left(-b/2a \, , \, \dfrac{4ac - b^2}{4a} \right)$ ✓

$$f(x) = ax^2 + bx + c$$

$$f(0) = c$$

$$f'(0) = b$$

$$f''(0) = 2a$$

So, $$\boxed{f(x) = \frac{f''(0)}{2} x^2 + f'(0) \cdot x + f(0)}$$